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MATHEMATICAL MODELING AND COMPUTER SIMULATION FOR DESIGNING MUNICIPAL REFUSE COLLECTION AND HAUL SERVICES

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solid waste management research grant no. UI-00699
to the Northwestern University was written by
S. WERSAN, J. QUON, *and* A. CHARNES
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FOREWORD

Current solid waste collection practices do not differ significantly from those used at the turn of the century. This lag in the development of new technologies has had a marked economic impact, because it costs four times as much to collect residential solid wastes and transport them to a disposal site as it does to actually dispose of them. Ways must be found, therefore, to maximize efficiency while minimizing costs.

An important objective of the U.S. Environmental Protection Agency is to aid in developing economic and efficient solid waste management practices. As authorized under the Solid Waste Disposal Act of 1965 (P.L. 89-272) and the Resource Recovery Act of 1970 (P.L. 91-512), the Agency awards research grants to nonprofit institutions in this effort to stimulate and accelerate the development of new or improved ways of handling the Nation's discarded solids. The present document reports on work completed under one of these research grants.

The first part of the report concentrates on the haul operation through a study of mathematical models for the location or selection of disposal sites and the allocation of collection territories thereto. The second part is concerned with an operational analysis of the collection and haul operations combined, done with the use of two computer programs especially written for the simulation of the operation of refuse systems.

Publication of this report does not signify that the contents necessarily reflect the views and policies of the U.S. Environmental Protection Agency, nor does mention of commercial products constitute endorsement or recommendation for use by the U.S. Government.

--SAMUEL HALE, JR.
Deputy Assistant Administrator
for Solid Waste Management

C O N T E N T S

Page

ABSTRACT xi

PREFACE 1

Section A 1

An Overview of Applications of System-Analytic Methods to Sanitary Engineering 1

Section B 6

The Nature of the Refuse System 6

Section C 10

General Outline of This Dissertation 10

PART I 1-1

On the Location of Refuse Disposal Sites and the Allocation of Service Territories 1-1

Chapter 1 1-2

Location Problems: A Review of the Literature 1-2

Chapter 2 1-19

Location-Allocation Models 1-19

Section A 1-19

Mathematical Definition and Assumptions 1-19

Section B 1-30

Solution of a One Sink Problem 1-30

Section C 1-34

Area Exclusions and the One Sink Problem 1-34

Section D 1-45

Multiple Sink Problems as Integer Programming Problems . . . 1-45

Section E 1-51

A Theory of Median Sets 1-51

Section F 1-64

An Approximating Algorithm Based on the Theory of Median Sets 1-64

Section G 1-73
Finding Initial Solutions 1-73
Chapter 3 1-78
Selection-Allocation Models 1-78
Section A 1-78
Mathematical Definition and Assumptions 1-78
Section B 1-81
Characterization as a Network Flow Model 1-81
Section C 1-88
An Approximating Algorithm: The Minimum Elimination-Gain Algorithm (MEGA) 1-88
Section D 1-97
Applications of MEGA to Location-Allocation and Other Problems 1-97
Chapter 4 1-98
Extensions of Selection-Allocation Models 1-98
Section A 1-98
S-A Problems with Weighted Sources 1-98
Section B 1-103
S-A Problems with Site Acquisition and Processing Costs . . 1-103
Section C 1-107
Extended Models for Considering the Development and Use of a Selected Site 1-107
Chapter 5 1-116
Summary and Conclusions 1-116

Figures

1-1 Median Solution and an Area Exclusion 1-35
1-2 Area Exclusion of a Central Business District 1-39
1-3 Structure of Constraint Matrix of (2.22) 1-50
1-4a Paths from Sources to Sinks Unavoidably Crossed 1-60
1-4b Reallocation to Cause Uncrossing 1-61
1-5 Initial Solution for Example 1 1-69
1-6 Result of First Exchange in Example 1 1-70
1-7 Result of Second Exchange in Example 1, Local Optimum Solution 1-71
1-8 Horizontal Problem Corresponding to Example 1, Figures 1-5 to 1-7 1-75

Page

1-9 Vertical Problem Corresponding to Example 1, Figures 1-5 to 1-7 1-75
1-10a P-System, N=7, S=4 1-82
1-10b A-R-System, S=4, m=2 1-82
1-11a Acceptance System Network for Extended Selection Model . 1-110
1-11b Physical Network of Extended Selection Model 1-111

Tables

1-1 Source Locations for Example 1 1-68
1-2 Comparison of Results ℓ_1 vs. E-metric 1-72
1-3 Coincidence Table for Horizontal and Vertical Solutions of Figures 1-8 and 1-9 1-77
1-4a Selection-Allocation Problem, N=7, S=5. Initial Allocation 1-90
1-4b Second Tableau, N=7, S=5 1-90
1-5 Illustration of Tie-Breaking 1-93
1-6a MEGA with Weights on a Problem of Maranzana. Initial Tableau 1-101
1-6b 7th Tableau 1-102
1-6c Final Tableau for M=2 1-102
PART II 2-1
On the Simulation of Refuse Collection and Haul Operations 2-1
Chapter 1 2-2
Introduction 2-2
Section A 2-2
Solid Wastes Literature 2-2
Section B 2-13
The Nature and Limitations of Simulation 2-13
Section C 2-33
The Simulation of Refuse Collection and Haul Operations . . 2-33
Chapter 2 2-36
Rational Methods for Design of Refuse Collection/Haul Systems 2-36
Section A 2-38
An Overview of the University of California Report 2-38

Page

Section B 2-44

Factors Influencing the Pickup and Haul Operations 2-44

Section C 2-50

Development and Use of Rational Design Formulas 2-50

Section D 2-62

An Evaluation of the University of California Report 2-62

Chapter 3 2-66

Development of the Simulation Program 2-66

Section A 2-66

Sources of Variability in Refuse Collection/Haul Operations 2-66

Section B 2-75

Some Concepts Underlying the Simulation Programs 2-75

Chapter 4 2-90

Simulation Studies of Refuse Collection and Haul under the Daily Route Method 2-90

Section A 2-93

Invariants of the Reference Systems 2-93

Section B 2-95

Variable Parameters of the Reference System 2-95

Section C 2-99

Results and Analysis 2-99

Appendix to Chapter 4: Notation 2-129

Chapter 5 2-132

Simulation Studies of Alternative Policies in a Constant Length Workday System 2-132

Section A 2-134

The Reference System under Constant Length Workday 2-134

Section B 2-138

Description of Alternate Policies 2-138

Section C 2-141

Results and Analysis 2-141

Appendix to Chapter 5: Notation 2-154

Page

Chapter 6 2-159

Extensions and Modifications of Simulation Models 2-159

Chapter 7 2-164

Summary and Conclusions 2-164

Figures

2-1 Flowchart for Simulating Buffon's Needle Experiment . . . 2-19

2-2 Flowchart of One Hand of the Game of "Clock" 2-22

2-3 Tree of Random Walk Moves 2-24

2-4 Probability Plot of Refuse Production, Winnetka, Illinois, March-April, 1958 2-68

2-5 Verbal Flowchart of the Movement of a Single Truck. Daily Route Method, Winnetka, Illinois 2-82

2-6 Verbal Flowchart of the Movement of a Single Truck. Constant Length Workday, Chicago, Illinois 2-85

2-7 Truck Capacity Utilization and the Number of Trips per Day vs. the Coefficient of Variability 2-105

2-8 Overall Collection Efficiency and Length of Workday vs. the Coefficient of Variability 2-106

2-9 Haul Efficiency and Haul Time vs. the Coefficient of Variability 2-107

2-10 Truck Capacity Utilization and the Number of Trips per Day vs. the Assignment 2-108

2-11 Overall Collection Efficiency and Length of Workday vs. the Assignment 2-109

2-12 Haul Efficiency and Haul Time vs. the Assignment 2-110

2-13 Overall Collection Efficiency and Length of Workday vs. One-way ℓ_1-metric Haul Distance 2-111

2-14 Haul Efficiency and Haul Time vs. One-way ℓ_1-metric Haul Distance 2-112

2-15 Correlation of Overall Collection Efficiency with the Pickup Coefficient 2-113

2-16 Queuing Problem at the Disposal Site: Waiting Time per Day vs. the Number of Unloading Docks 2-114

2-17 Hypothetical Collection Area 2-137

Page

Tables

2-1 Statistics on Refuse Production. Winnetka, Illinois, March-April, 1958 2-69

2-2 Values of Input Parameters 2-96

2-3 Example Output Information from Simulation 2-100

2-4a Distribution of Percent of Trips vs. Percent Capacity Used. Coefficient of Variability $\sigma/\mu = 0.1$ 2-102

2-4b Distribution of Percent of Trips vs. Percent Capacity Used. Coefficient of Variability $\sigma/\mu = 0.3$ and 0.6 . 2-103

2-4c Contingency Table Analysis of E_c Frequency Histogram--$A = 2.04$, $\sigma/\mu = 0.1$ 2-104

2-5 Frequency Distribution of Length of Workday and Settings of Input Values of 25 Cases 2-119

2-6 Cumulative Distribution of E_c for $\sigma/\mu = 0.3$, 0.6 with $A = 3.06$ 2-122

2-7 Summary of Values of Input Parameters 2-136

2-8 Summary of Operational Policies Considered 2-139

2-9 Example Output Information from Simulation Model 2-142

2-10 Example Output Histogram of Length of Workday 2-143

2-11 Example Output Histogram of Truck Capacity Utilization . 2-144

2-12 Effect of Relay Policy on the Quality and Cost of Service Provided 2-146

2-13 Effect of Overtime Policy on the Quality and Cost of Service Provided 2-148

2-14 Effect of Assignment Policy on the Quality and Cost of Service Provided 2-150

2-15 Cost and Quality of Service for Twelve Operational Policies 2-151

Appendix 2-A 2-171

Figures

A.1 Haul Speed vs. Round Trip Distance 2-179

A.2 Verbal Flowchart of Outer Program 2-184

A.3 Verbal Flowchart of Event Clock 2-186

A.4 PUT-Subroutine for Storing Events 2-189

A.5 Event Type 1: Truck Arrives at Disposal Site 2-191

A.6 Event Type 2: Collection Unit Entered for First Time. 2-194

A.7 Event Type 3: Truck Leaves Disposal Site 2-196

Page

A.8 Event Type 4: Collection Unit is PROP(H) Finished . . 2-199

A.9 Event Type 5: Partially Done Collection Unit is Entered 2-201

A.10 Event Type 6: Truck Arrives at Garage 2-203

A.11 CMPJOB: Subroutine to Determine Refuse Amount and Collection Time
TRATIM: Subroutine to Compute Haul Time and Distant . 2-204

A.12 CAPRUL: Capacity Rule Subroutine
FINEXU: Find Next CU Subroutine 2-209

A.13 HISTOG: General Histogram Reader 2-212

A.14 SOUP: General Statistics Accumulator 2-214

A.15 PWFHST: Frequency Histogram Packer 2-216

A.16 UNPWFH: Frequency Histogram Unpacker 2-219

Tables

A.1 Example of Input Card Deck 2-235

A.2a Sample of Printed Output, First Page 2-237

A.2b Sample of Printed Output, Second Page 2-238

A.3a Listing of Punched Card Output 2-243

A.3b Listing of Punched Card Output (Cont.) 2-244

REFERENCES 2-282

ABSTRACT

Part I of this dissertation begins with consideration of problems called "location-allocation" problems in which several disposal sites are to be physically located with simultaneous allocation of refuse sources to disposal sites. The disposal site may be located anywhere in a plane and the measure of effectiveness is minimization of aggregate haul distance. By measuring distance with the ℓ_1-metric three things are accomplished:

(1) It is possible to reduce these ℓ_1-metric location-allocation problems to mixed integer programming problems.

(2) Using the mixed integer programming formulation, one may add constraints which proscribe certain areas for the placement of disposal sites.

(3) The single site location problem has a closed-form solution related to the geometric median of the source points. Using this property, a Theory of Median Sets is established and an alternating location-allocation algorithm is constructed for the multi-site problem.

A second class of problems called "selection-allocation" problems is defined. Here the set of eligible disposal site locations is known and one is to pick a proper subset of these and a refuse source allocation which minimizes the aggregate haul distance or cost. These problems are characterized as mixed integer programming problems with a coupled network analog and an approximating algorithm, the Minimum Elimination Gain Algorithm is established. This latter algorithm is extended to consider weighted sources and site acquisition costs. The network analog is extended to consider site acquisition, facility development and operation costs as well as aggregate haul cost.

Part II of this dissertation traces the development of two computer simulation models. The first model is based on the daily route method of refuse collection practiced in the Village of Winnetka, Illinois. Using data relevant to Winnetka, a series of simulation runs was made to delineate the interdependencies of parameters involved in the functioning of a refuse collection system. In particular, the percent of truck capacity utilized, the number of daily trips, the overall collection efficiency, the length of workday, the haul efficiency, and the haul time as--percentage of total time were measured as functions of the coefficient of variability of refuse production, the refuse assignment and one-way haul distance. A further set of runs studied the effect of the number of unloading platforms on average and maximum waiting times at the disposal site. The interpretation of the results presented in tables and graphs allows numerical bounds on the usefulness of the daily route method to be established regarding the coefficient of variability, the refuse assignment and haul distance.

The second model is based on constant length workday rules found in Chicago, Illinois. Using data relevant to Chicago, a series of runs was made to measure the quality of service and cost effectiveness of different combinations of overtime, last-load relay and assignment policy. Here assignment means both the average daily number of truck loads and the average daily number of eight hour shifts. The results presented in a series of tables show as one example that the use of overtime is most cost effective when combined with a reasonable time assignment (less than one).

PREFACE

Section A

AN OVERVIEW OF APPLICATIONS OF SYSTEM-ANALYTIC METHODS TO SANITARY ENGINEERING

The past decade has seen a dramatic growth of awareness and concern in the general public about the environments that surround us. Public apathy toward the pollution of our rivers and the poisoning of our atmosphere is gone. We have become concerned about rebuilding our cities, beautifying our roadsides, preserving our remaining wilderness areas, and showing that the twentieth century and de Tocqueville's America [Pr. 15][1/] are not wholly incompatible.

This movement is a natural outgrowth of the conservation movement which was born in the early years of this century and is a reaction to our 40% increase in population (1939-1964) and our increasing urbanization, suburbanization, and "automobilification" .

In the growth of this awareness, matters which were once universally ignored became the crusades of a foresighted few, then the expected duties of local and state governments, and finally, of the federal government

---

[1/] Numerals in brackets indicate references. See page 2-282.

which today aids local governments in the construction of sewage treatment facilities and sponsors research to improve the technologies and augment the basic scientific foundations of the disciplines involved. One of the research philosophies and bodies of methodology that have been adopted in some of this research is the "systems" approach. As expressed by W. R. Lynn [Pr. 11]:

> "It is obvious that satisfactory solutions to this problem [disposition of solid wastes] cannot be obtained by viewing each operation within the processes of refuse collection as an independent unrelated function. Rather, in order to achieve efficient solutions to these problems effort must be directed toward viewing the problem in its entirety as an interconnected system of component operations and functions.
>
> Unfortunately, until comparatively recently, solutions to this class problem have been unavailable because of the difficulty in making a systematic analysis of large complex problems. However, the advent of new mathematical techniques in systems analysis and operations research coupled with the availability and facility of high speed computing devices will now permit investigation of this type."

In actual practice, applications of these techniques to the field of sanitary engineering have been pursued in two directions. These are: To better understand and design component processes and to synthesize such optimal components into optimal systems. The aim of this dissertation is to follow both of these trends in that field of sanitary engineering which is concerned with the disposal of solid wastes, but it is not irrelevant to review briefly the results that have been accomplished in other areas of sanitary engineering by the use of those methods. It will be seen that these results are largely concentrated in the area of water pollution and the closely related area of water resources management.

W. S. Galler [Pr. 3] (see also [Pr. 4]) used nonlinear programming to determine the optimal configuration (radius, depth, and recirculation ratio) of biological (trickling) filters, an important sewage and industrial wastes treatment process. By optimal was meant minimum operating and construction cost subject to the constraint that the pollutional load (biochemical oxygen demand or BOD) be reduced to a prescribed level.

Dr. Lynn's dissertation [Pr. 10] (see also [Pr. 8] and [Pr. 9])represents a middle ground proceeding in the direction of increasing synthesis. Assuming the existence of optimally designed components capable of operating at prescribed levels, Lynn constructed a mathematical model of the flow of sewage through a treatment plant. The object was to select the types of treatment processes, their operating levels, and their order of application to achieve for a treatment plant as a whole the same objective stated above regarding the biological filter. A second, dynamic model investigated the elements of timing and financing in the face of population growth.

Proceeding further in the direction of synthesis are four works which deal with the pollution problems and the water resources management of entire river basins.

A 1962 book of Maass, et.al. entitled Design of Water Resources Systems [Pr. 12] discussed the theoretical, methodological, economic, and political foundations underlying such system designs. No specific river system was analyzed.

This book laid the groundwork for the later work of Hufschmidt and Fiering [Pr. 5], Simulation Techniques for Design of Water Resources Systems, in which computer simulation techniques were used to determine the optimal placement of six multipurpose dams on the Lehigh River in Pennsylvania.

Linear programming models of pollution abatement in an entire but largely hypothetical river basin were constructed by R. A. Deininger [Pr. 2]. One such model sought minimal treatment cost subject to prescribed levels of BOD reduction. A second model incorporated a dual criterion: BOD to be reduced and dissolved oxygen to be augmented. Several other models were formulated but not pursued. Perhaps the most interesting of these is a model which considers the cost of water purification as well as sewage treatment on the realistic premise that the river is used subsequently downstream for water supply.

A later work by Loucks [Pr. 6] (see also [Pr. 7]) analyzed the cost and effectiveness of treatment from the viewpoint of probability theory. The aim here was to determine the cost of maintaining a prescribed probability level on the minimum dissolved oxygen content of a river under the random variation in pollutional load and other relevant factors.

A discussion of published work and work in progress on the application of system analytic methods to the field of solid wastes will be deferred until the introduction to Part II, i.e. Chapter 1.

Section B

THE NATURE OF THE REFUSE SYSTEM

It is appropriate and logical at this point to develop some feeling for the nature of refuse systems and the complex inter-relation of its parts that impels the system analytic approach.

Aesthetic and economic considerations **aside**, a city is — to one responsible for the planning and operation of a municipal refuse collection service — a conglomeration of people together with their residences, businesses, factories, and institutions set down in a maze of streets, highways, and traffic. In the normal course of events, these people produce residential, commercial, manufacturing, and institutional wastes that must be gathered by men and equipment, transported over streets and highways by men and equipment, and finally disposed of by some means, again by men and equipment.

Overlying and governing these three major areas of collection, transport and disposal is the system of policy, planning, finance, and administration that pays the men, buys the equipment, acquires the disposal facilities, creates an intelligent working entity from all of these, and acts as a buffer between those served and those serving.

The preceding has been a bare physical and institutional outline of a refuse system. We may flesh-out this skeleton by noting that a refuse system functions in the environment of a particular city: This city has its own particular climate, topography, seasonal variations, growth patterns, and population with its own habits, traditions, standards of living, and political institutions — all of which affect a refuse system. Moreover, a refuse system does not stand still in time. Long term trends such as the increased use of prepared and convenience foods reduce the putrescible organic content of refuse, but yield more containers to throw away while the introduction of garbage grinders produces similar changes.

The most important driving force behind a refuse system is its workload which is largely determined by the following set of decisions:

(1) Classes of residences and other sites to be served, e.g.,are businesses required to hire a private hauler?

(2) The type(s) of refuse to be collected, e.g. are grinders to be used? Shall yard refuse and leaves be accepted?

(3) The frequency of collection. This may depend on the season and the amount of putrescible matter present.

(4) The degree of cooperation required from those served, more specifically with regard to

   (a) Separation of refuse types into different containers,

   (b) Yard, alley or curb pickup, and

   (c) Standardization of container size.

Items (1), (2), and (3) have the most profound effects on the entire system for they dictate via the workload the numbers of men and the cubic yards of truck capacity that will be required. Item (4) enters the picture largely by affecting the efficiency of collection as measured for example in man-minutes per ton of refuse collected.

The amounts of refuse, their types, and the degree of separation may also affect decisions on the best method of ultimate disposal. The method of disposal, in turn, bears heavily on where the disposal site(s) may be located, and the location of the disposal site(s) determines the proportion of the workday spent on the haul operation. Finally, the length of the haul operation is a significant factor in setting work force size and purchasing equipment.

Thus, there is here truly a system which eventually must be viewed "... in its entirety as an interconnected system of operations and functions". [Pr. 11]

Section C

GENERAL OUTLINE OF THIS DISSERTATION

This dissertation is divided into two distinct parts. In the first part we isolate and concentrate on the haul operation through a study of mathematical models for the location or selection of disposal sites and the allocation of collection territories thereto. The second part of this dissertation is concerned with an operational analysis of the collection and haul operations combined and this is done with the use of two especially written computer programs for the simulation of the operation of refuse systems.

Among the long-term secular changes which cities face is the urbanization of the land separating them; much has been written lately about the growth of "megalopoli". This has two effects on the planning of refuse disposal. First, it means that the land required for disposal operations is becoming more expensive, or alternatively land at the right price is retreating further from the city. Secondly, it means that the component governmental units of megalopoli must sooner or later join together in a regional approach to their common problem. As expressed by the Northeastern Illinois Metropolitan Planning Commission:

> "That refuse disposal is a metropolitan problem is evident in the frequent shipment of refuse across county and municipal boundaries. Large tracts of land are needed to accomodate refuse, and the difficulty of obtaining such tracts often requires refuse disposal sites to be located far from the collection sources." [Pr. 13]

For economic perspective we may note that the haul operation[1] accounts for some 25-35% [Pr. 1] of municipal refuse budgets and these aggregate to some four-billion dollars a year nationally.

Thus motivated, we are led to investigate the questions:

(1) Where is the best place to locate one or more refuse disposal sites given the locations of the areas to be served?

(2) Which areas should be served by which disposal site?

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[1] Roughly, the haul operation may be defined as that part of the refuse collection-disposal activity which takes place between the last container of a load and the first container of the next load. It is here extended to include travel to and from the garage.

The assumption lying behind these questions is that we are free to place a disposal site anywhere in a large, unrestricted area. The discussion above makes it apparent that this is an unrealistic assumption. Hence, building on the insights gained from answering questions (1) and (2) we next ask:

(3) What happens to the choices of location and allocation of collection sources when certain areas are ineligible for the location of disposal sites?

It is clear that we get even closer to reality when the process of declaring certain areas ineligible culminates in the situation wherein only a small number of land parcels remain eligible. Now we ask:

(4) Given a definite number of eligible sites and their physical locations, what is the best way to pick a proper subset[2] of them to be disposal sites and to allocate collection sources to them?

The mathematical models that yield answers to these questions are investigated in Part I of this dissertation. The models arising from questions (1), (2), and (3) are termed location-allocation problems and are discussed in Chapters 1 and 2 . The models arising from question (4) are termed *selection-allocation* problems and are discussed in Chapters 1 , 3 and 4 . The selection-allocation model lends itself readily to considerations such as *site acquisition cost* and *processing costs* (per unit of refuse processed) at the selected sites. Selection-allocation models embodying these features are considered in Chapter 4 .

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[2] Meaning some but not all. We shall later restrict this to mean also ≥ 2 .

The matter of processing cost naturally generates a more general question:

> (5) Having selected sites on the basis of reasonable system-wide transportation and acquisition costs, *what disposal processes* should be used at these sites?

In Chapter 4 models to answer this question are discussed, it being pointed out that mature engineering judgment has already limited the set of eligible sites to those suitable to the disposal processes deemed feasible on the basis of other considerations, thus this sort of model is really concerned with *choices of capacity and processing rate*. In such models we are presented with a total amount of refuse to be processed and the points where the refuse originates. This refuse is to be taken to a certain number of sites selected from a list of eligible sites at "minimum" aggregate transit cost. For each site selected we pay a one-time acquisition cost, and we select a processing capacity and rate from a list of alternatives appropriate to the given site. This choice requires the payment of a one-time development cost, and imposes on us a capacity and processing rate which when taken together with those from all of the other sites must be adequate to process the refuse on hand. It is to be noted that the total cost is made up of transit, site-acquisition, site-development, and operation costs (suitably discounted for time disparities) and thus the overall optimum program may not really have minimal transit cost which was the only criterion in the first of these models.

It is clear upon reflecting on this sequence of increasingly complex models that further increases in complexity buy very little further in predictive powers and insight. The chief criticism, might be phrased as follows:

The methods of Part I concentrate heavy guns upon a frozen sliver of time taken from a world undergoing short term random variation, seasonal fluctuation, and long-term (secular) changes. How can methods which study static situations be used to understand a dynamic reality?

Or more fundamentally, what methods are available to study dynamic systems? To answer these questions we submit the following brief list of methods:

1. Empirical Approach: In the context of refuse collection and haul systems, this means study of dynamics in the field by experimenting with actual crews, equipment, etc.
2. Analogy: Planning is done by comparison of a given city to another and using an analogous solution.
3. Mathematical Analysis: An example of such analysis is contained in the models of Part I. Others will be discussed in Part II.
4. Simulated Experimentation: In this age, this generally means a simulated experiment conducted by the use of a computer program.

Part II of this dissertation is concerned with the design, use, and interpretation of results from such a computer simulation model hence we defer until Chapter 1 , a more detailed discussion of the nature and limitations of simulation and its proper place with respect to the other methodologies just listed.

In Chapter 2 we enlarge upon certain rational methods which have been developed for the design of entire static refuse collection/haul systems. Chapter 3 first examines the sources of variability selected as being of greatest importance in a refuse system and then turns to a discussion of some of the concepts used to render the real world physical entities of men, trucks, and refuse amenable to simulation by a digital computer program. Two programs are described here. The first simulates the daily route method of refuse collection as practiced in the Village of Winnetka, Illinois. The description of this program is supplemented by a detailed description in Appendix 2-A . The second program, derived from the first, simulates the fixed-length workday method of refuse collection as practiced in certain parts of the City of Chicago. Only the specific points of difference are described.

Chapters 4 and 5 present the main results of Part II. They are concerned with the results obtained from using the two simulation models on particular subsets of input data. Each chapter discusses the inputs used, the results obtained, and the practical implications of these results to the planners and operators of refuse collection/haul systems.

Chapter 6 proposes extensions of the simulation models and draws upon shortcomings discovered therein to delineate characteristics desirable in future models. Finally, Chapter 7 summarizes and states the results of Part II.

PART I

ON THE LOCATION OF REFUSE DISPOSAL SITES AND THE ALLOCATION OF SERVICE TERRITORIES

CHAPTER 1

Section 1

Location Problems: A Review of the Literature

The location-allocation and selection-allocation models to be investigated in Part I have their roots in mathematical antiquity. The germinal problem for the location-allocation model is the generalized Weber problem discussed immediately below. Selection-allocation problems arise out of location-allocation problems in the manner suggested in Section C of the Preface.

The generalized Weber problem is

Given a set of fixed points $\{(a_i, b_i), i = 1, 2, \ldots, n\}$ and weights $\{w_i > 0\}$, determine (x, y) that minimizes

$$F(x, y) = \sum_{i=1}^{i=n} w_i[(x - a_i)^2 + (y - b_i)^2]^{1/2} \qquad (1.1)$$

i.e. the weighted sum of the distances from the fixed points to the minimum point.

Up until 1962, the only things known about the solution to (1.1) were: a) for special values of n $(n \leq 4)$; b) for certain degenerate cases such as colinearity [all the points on a straight line yielding $x = \sum_i w_i a_i / \sum_i w_i$, $y = \sum_i w_i b_i / \sum_i w_i$ (the means)] and n-fold symmetry of the (a_i, b_i) and the w_i [e.g., the apices of an n-gon where n is even and opposite apices have equal weights ⇒(x, y) at center of n-gon] ; c) for general n , the theorems of Tedenat and Steiner [A 5] which cover the case of equal weights.

Tedenat's Theorem (1810): A necessary condition that (x, y) be the minimum points of (1.1) with all w_i equal is that the sum of the cosines of the angles between any arbitrary line in the plane and the set of lines connecting (a_i, b_i) to (x, y) be zero.

Steiner's Theorem (1837): A necessary and sufficient condition that (x, y) be the minimum point of (1.1) with all w_i equal is that the sum of the cosines and the sines of said angles be zero. (See the reviews of results in Cooper [A 5] and Eisemann [A 10].)

Alfred Weber's classic Theory of the Location of Industries [A 13] (reviewed in [A 5]) is apparently the first place that the problem appears in an economic context as opposed to a pure mathematical abstraction. The economic context makes natural the first apparent introduction of weights into the problem (hence the name), although one wonders if the weighted problem may not be buried in the early literature of physics inasmuch as the weighted (or equal weights) situation is easily represented by a physical analog which Weber and several others since have done (see [A 10] and [A 27]).

It should be noted first that the problem is not easily amenable to solution by standard methods of elementary calculus, and except for the cases $n \leq 4$ the results cited above are characterizations of the solution not directions for educing it. Secondly, and sadly, one has the feeling that articles concerning this problem (1.1) will appear in the literature until the end of time, each with scant reference to what has gone before. For example, it stretches credulity beyond bounds to believe that no one had proved a theorem on existence and uniqueness prior to 1961 (see Palermo [A 27] and Tideman [A 30]).

The first reference (1962) to an algorithm for the solution of (1.1) is found in Kuhn and Kuenne [A 22] . A second algorithm (1963) is that of Cooper [A 5] which we discuss in detail.

Having proved the existence of a minimum (but not uniqueness), i.e.

$$\frac{\partial^2 F}{\partial x^2} > 0, \ \frac{\partial^2 F}{\partial y^2} > 0$$

and the Lagrange condition

$$\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - \left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 > 0 \ ,$$

Cooper introduces the following recursive scheme: Let $D_1^k = [(x^k - a_1)^2 + (y^k - b_1)^2]^{1/2}$ where k denotes steps of the recursion process.

Then differentiating (1.1) and solving for x , y we have

$$\left.\begin{aligned}
x^{k+1} &= \sum_i (w_i\ a_i/D_i^k)\Big/\sum_i (w_i/D_i^k) \\
y^{k+1} &= \sum_i (w_i\ b_i/D_i^k)\Big/\sum_i (w_i/D_i^k) \\
\text{with} & \\
& \left.\begin{aligned}
x^o &= \sum_i w_i\ a_i\Big/\sum_i w_i \\
y^o &= \sum_i w_i\ b_i\Big/\sum_i w_i
\end{aligned}\right\}\begin{array}{l}\text{colinear}\\ \text{solution}\end{array}.
\end{aligned}\right\} \cdot (1.2)$$

The iteration is maintained until

$$|x^{k+1} - x^k| < \varepsilon \text{ and } |y^{k+1} - y^k| < \varepsilon .$$

No proof of convergence is given. Since the solution is unique (in the absence of colinearity) one need worry only whether $\{|x^k|\} \to \infty$ or whether it might oscillate. Cooper has had no such ill fortune in hundreds of uses of this algorithm reported here and in a subsequent article [A 6] .

Finally, Bellman (1965) reduced the problem to a dynamic programming problem [A 2] .

As a last interesting note on (1.1), it should be noted that it may be converted into an unweighted problem simply by replication of fixed points, i.e.

Let $M > 0$ be $\ni Mw_i$ is an integer $\forall i$ and let $N = \sum_i Mw_i$, then

$$\text{Min } F(x, y) = \frac{1}{M} \text{Min } G(x, y)$$

where (1.3)

$$G(x, y) = \sum_{i=1}^{i=N} [(x - a_i)^2 + (y - b_i)^2]^{1/2}$$

is equivalent to the original problem.

We turn now to the thither side of the hyphen, allocation. To understand this we shall first cite several examples of location-allocation problems and then adopt some terminology which will be used consistently throughout Part I.

Location-allocation problems arise in every social and economic context where an activity is to be operated on such a universal scale that it is neither possible nor desirable to have the center of this activity at a single location. The classic example in the area of location economics is the choice of manufacturing sites to satisfy the demands of a population distributed over a plane, i.e. the problem of determining "market areas". Other examples are the placement and division of network among telephone exchanges, the location of schools and the setting of school district boundaries, and from electrical engineering, the placement of electronic modules in the backplane of a computer to minimize wire length. Our present example is, of course, the determination of refuse disposal sites and the allocation of collection territories thereto.

To keep in step with this purpose we adopt the following terminology: we shall call the fixed points the "sources" (of refuse) and the minimal points "sinks". (This is the reverse of Cooper's [A 5], [A 6] terminology as his work is based on a distributive rather than an ingathering process.) We shall, furthermore, coin the term "sinkmates" or "sinkmate groups" when we are speaking of sources allocated to a given sink. Thus, using this terminology, we are able to say that location-allocation problems are concerned simultaneously with the optimal location of sinks and the optimal determination of sink-mate groups. A more exact mathematical characterization is deferred to Chapter 1.

The earliest work in this area was that of L. Kantorovitch, the distinguished Russian mathematician, in 1942, but not available in English translation until 1958 [A 14] . This is a highly abstract mathematical discussion, the full complexity of which, is beyond our present scope to discuss. We shall, however, approach it by citing an example with an infinite number of sources and an infinite number of sinks whose solution is characterized by this paper. The characterization covers also the finite source - finite sink problems with which we shall be concerned.

Kantorovitch's example is as follows:

Suppose we have the topographical or relief map of a certain region R before and after a cut and fill operation, i.e.

$z = f_b(x, y)$ is the relief before

$z = f_a(x, y)$ is the relief after .

Assuming conservation of volume

$$\iint_R f_b(x, y)\, dx\, dy = \iint_R f_a(x, y)\, dx\, dy ,$$

and given a continous function $C(x, y, x', y') \geq 0$ which is the cost of moving a unit volume from (x, y) to (x', y') , find a translocation function $\psi_o(x, y, x', y')$ which tells how much earth to move from (x, y) to (x', y') which minimizes the total earth moving expense.

The gist of Kantorovitch's characterization is based on the following definition: A translocation is said to be potential if there exists a function $U(x, y)$ such that

$$|U(x, y) - U(x', y')| \leq C(x, y, x', y')$$

and

$$U(x, y) - U(x', y') = C(x, y, x', y')$$

when

$$(x, y) \rightarrow (x', y').$$

Kantorovitch then proves that a translocation is minimal if and only if it is potential.

The identical result was rediscovered and published as an abstract in 1962 by McIlroy [A 23] . The underlying assumptions and method of proof[3] are nowhere nearly as general as those of Kantorovitch, but to be fair, McIlroy does mention having produced a solution algorithm.

Kantorovitch's work also anticipates the potential characterization of W. Prager [A 28] writing before Kantorvitch's work was translated.

In Isard's well known Location and Space-Economy [A 18] (also reviewed in [A 5])especially Chapters 7 and 11, he considers and formulates problems concerned with many sources (our sinks) and destinations (our sources), but actually confines his attention to marketing areas each served by its own source. This is, as we shall see, the equivalent of several location problems of the Weber type discussed above and ducks the issue of simultaneous determination of both location and allocation.

For other general and specialized works in location theory and application see Marshak [A 15] and the comprehensive bibliography [D 2] .

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[3] As understood from attending the symposium where the paper was given.

Cooper's 1963 paper entitled Location Allocation Problems (op. cit.) tackles the problem of simultaneous determination of location and allocation by letting the source locations themselves represent a set of eligible sink locations. This yields $\binom{n}{m} = \frac{n!}{m!(n-m)!}$ solutions to be evaluated as follows:

1) Assign each of the $n - m$ non-sink sources to the nearest sink.
2) Sum the distances.

When this has been done the smallest such allocation, or some set of the smallest such allocation can be refined by the "exact location method" of (1.2) . We note as does Cooper that this procedure does not guarantee optimality and although time consuming (since $\binom{n}{m}$ can be a very large number) is preferable to complete enumeration of sinkmate groups since $S(n, m) = \frac{1}{m!} \sum_{k=o}^{k=m} \binom{m}{k}(-1)^k(m-k)^n$ is much larger.

To this we add our notes that this method of procedure renders the problem a selection-allocation problem, i.e.,the sinks are to be chosen from amongst a set of known points. Hence the methods of Chapter 3 are applicable as will be shown. Second,we note that allowing sinks to be located at sources will render some of the D_i's equal to zero. In a personal communication Dr.

Cooper has explained that terms for which $D_1^k = 0$ are to be dropped from the summations of (1.2) .

In a subsequent article, Cooper [A 6] provided theoretical bounds on the value of the objective function F through the following theorems:

Theorem 1 (Cooper): For the n source, one sink location (-allocation) problem

$$\frac{1}{n-1} \sum_{k=2}^{k=n} \sum_{\ell=1}^{\ell=k-1} d_{k\ell} \leq \text{Min } F$$

where

$$d_{k\ell} = [(a_k - a_\ell)^2 + (b_k - b_\ell)^2]^{1/2} .$$

Theorem 2 (Cooper): For the n source, m sink location-allocation problem if there are at least three sources in each sinkmate group and $e_1 \leq e_2 \leq e_3 \leq \cdots \leq e_{m\,n}$ are the factors $d_{k\ell}$ (see theorem 1) after being ranked then

$$\frac{1}{2} \sum_{p=1}^{p=n} e_p \leq \text{Min } F .$$

Theorem 3 (Cooper):

$$\sum_{i=1}^{i=n} [(a_i - \overline{x})^2 + (b_i - \overline{y})^2]^{1/2} \geq \text{Min } F$$

where

$$\overline{x} = \sum_{i=1}^{i=n} a_i/n \,, \; \overline{y} = \sum_{i=1}^{i=n} b_i/n \;.$$

Further, he described and compared the following heuristic algorithms:

A. Destination Subset Algorithms: This is the algorithm of the first article.

B. Random Destination Algorithm: This is algorithm A except that the sink subsets are generated randomly on the supposition that luck may get us to or near the optimum sooner than total enumeration of the $\binom{n}{m}$ possibilities.

C. Successive Approximation Algorithm: In this algorithm one solves first the two sink problem by algorithm A. We next place a third sink at each one of the other sources choosing thus the best one. This is repeated for a fourth sink, etc., until the requisite number of sinks m is reached.

D. Alternate Location and Allocation Algorithm: Starting with an initial allocation, the "exact location method" of (1.2) is used to locate the sinks. Then each source is examined to see if it is nearer any other sink than its own. If not, we are done. If the allocation changes, the process is repeated.

Cooper notes that if D, R, A are the sets of solutions generated by algorithms A, B, and D respectively and if s is the single solution generated by algorithm C then $R \subset D$, $s \in D$ but $D \cap A = \emptyset$, $R \cap A = \emptyset$ and $s \notin A$.

When algorithms A and B are supplemented by refining the best solutions by the "exact location method of (1.2) it is found that algorithm A is most accurate but by far slowest. Algorithms B and D are tied for accuracy with D, three to four times as fast.

In support of the notion that economically a very good solution may be as good as the optimal solution, Cooper adduces evidence to the relative insensitivity of F to large changes in the sink locations with correct or nearly correct allocations, i.e.,location-allocation problems are typified by having many solutions close to the optimum.

Selection-allocation problems have been touched on briefly above and in section C of the Preface. The nearest approach to a problem of this sort in the literature is in the work of Hakimi [A 15], Frank [A 12], and Maranzana [A 24] . Actually, Hakimi and Frank's work stands between the area selection problems discussed above and selection-allocation problems where the eligible sinks are a set of known points. In these works, the minimum point must be a point lying on a network of paths connecting weighted vertices.

For Hakimi, the weights are known while for Frank the weights are probablistic, but in neither work is the problem of simultaneous sink selection and allocation considered. Maranzana [op. cit.] poses a problem which is actually a selection-allocation problem. Define P_j as a center of gravity (C.G.) of a set of weighted nodes Q by:

$$\sum_{P_k \epsilon Q} w_k \, D(P_j, P_k) \leq \sum_{P_k \epsilon Q} w_k \, D(P_i, P_k) \quad \text{all } i \qquad (1.4)$$

where $D(P_j, P_k)$ is the minimal distance on network paths from P_j to P_k .

The algorithm proceeds essentially as Cooper's algorithm D with the C.G. (1.4) replacing the "exact location method" (1.2). The method is monotonic but can be shown to converge occasionally to a local optimum which is not a global optimum (see Chapter 4).

For other works of related interest see Efroymson and Ray [A 8], Kuhn and Kuenne [op. cit.], Armour and Buffa [A 1], Garett and Plyter [A 14], Kuehn and Hamburger [A 21], Baumol and Wolfe [A 3] , Koopmans and Beckmann [A 20], Eilon and Deziel [A 9], Hillier [A 17] , and Drysdale and Sandiford [A 7]. A elementary, tutorial approach to several kinds of location problems is provided by the second chapter of Noble's recent book [A 26].

A selection of the electrical engineering literature pertaining to the backboard wiring problems is included in the bibliography for Part I (section B).

The approach taken in this dissertation to location-allocation problems is two-fold. The Euclidean metric (distance function) $\sqrt{(x - a_i)^2 + (y - b_i)^2}$ is replaced by the ℓ_1-metric variously termed rectangular distance or the "metropolitan" metric wherein the distance from (x, y) to (a_i, b_i) is given by $|x - a_i| + |y - b_i|$. This is a valid approach in its own right as well as an excellent approximation to the Euclidean distance problems of Cooper. This altered objective allows the solution of the pure location problems to be stated in closed form rather than as a recursive algorithm and permits a particularly efficient version of Cooper's algorithm D to be constructed. It also permits the resultant location-allocation problem to be expressed as a linear program in integer variables.

The first use of the ℓ_1-metric in such problems is due to Prager [A 28]. The median approach was first recognized by Charnes, Quon and Wersan [A 4] , [A 34] and subsequently rediscovered by Francis [A 11]. See also Hilliar [17].

The approach presented here to selection-location problems is felt to be unique and without precedent in the literature.

CHAPTER 2

LOCATION-ALLOCATION MODELS

Section A

Mathematical Definition and Assumptions

Location-allocation problems in their most general form are typified by the following statement:

Given: (1) The location in Cartesian co-ordinates of n points called sources. Note that the dimensionality of the space containing these points need not be specified although we assume two dimensions hereafter.

(2) The amount of a homogeneous commodity of flow present at each source.

(3) A number $m < n$ of points called sinks. The determination of m is touched upon by Cooper [A 5]. This is a matter of economic trade-off analysis between the saving in transportation costs resulting from many sinks and the capital and operating expenses incurred in obtaining them. See Chapter 4.

(4) Either

(i) A set of n functions each giving the cost of moving a unit of the commodity from the i-th source to a point (x, y) .

or

(ii) A set of m functions each giving the cost of moving a unit of the commodity from an arbitrary point x, y to the j-th sink.

The distinction between these alternate schemes is that the latter is the more natural formulation if the cost depends on the total amount received at the j-th sink.

(5) A set of capacity constraints which limit the amount of commodity each sink can or may receive. Likewise, there may be floors under these amounts.

To Determine:

(1) The location in Cartesian co-ordinates of the m sinks (location variables).

(2) The amount sent from the i-th source to the j-th sink, and thereby, the total amount received at each sink (allocation variables).

Such that: The aggregate weighted transportation cost is minimized.

It is obvious that very little can be done with so general a formulation until some specification and simplification is done. The first such choice we make is to assume

(A1) Unit transportation cost shall be independent of the amount received at the j-th sink. This is a reasonable assumption in the context of refuse disposal sites. This may not be so for the processing costs thereat which are not considered in this chapter.

In terms of the above,

Let:

$$\left.\begin{array}{ll} (a_i,\ b_i) & \text{be the location of the sources} \\ V_i > 0 & \text{be the commodity present at} \\ & \text{the i-th source} \end{array}\right\} i = 1,2,\ldots,n$$

$$\left.\begin{array}{ll} (x_j,\ y_j) & \text{be the locations of the sinks} \\ L_j,\ U_j & \text{be the lower and upper capacity} \\ & \text{bounds at the sinks} \end{array}\right\} j = 1,2,\ldots,m$$

$$\left.\begin{array}{ll} v_{ij} & \text{be the allocation variables} \\ \psi_{ij} \equiv \psi(a_i,\, b_i,\, x_j,\, y_j) & \text{be the cost} \\ & \qquad \text{functions} \end{array}\right\} \quad \begin{array}{l} i = 1,2,\ldots,n \\ j = 1,2,\ldots,m \end{array}$$

Then the problem becomes

$$\left.\begin{array}{ll} \text{Minimize} & \displaystyle\sum_{i=1}^{i=n} \sum_{j=1}^{j=m} v_{ij}\ \psi_{ij} \\ \\ \text{Subject to (S.T.)} & \displaystyle\sum_{j=1}^{j=m} v_{ij} = V_i \\ & \text{(conservation)} \\ & \displaystyle L_j \leq \sum_{i=1}^{i=n} v_{ij} \leq U_j \\ & \text{(capacity)} \\ \text{and} & v_{ij} \geq 0\ \forall (i,\, j)\ . \end{array}\right\} \tag{2.1}$$

Theorem 1: In the absence of capacity constraints, there is an optimal solution of (2.1) obtained by sending all of the commodity V_i at a source to the costwise nearest sink.

Proof: Let the optimal solution of (2.1) without the capacity constraints be given by

$$\{v_{ij}^* : v_{ij}^* \geq 0, \sum_{j=1}^{j=m} v_{ij}^* = V_i\} \ , \{(x_j^*, y_j^*)\}$$

and let $\psi_{ij}^* = \psi(a_i, b_i, x_j^*, y_j^*)$. Then

$$\sum_i \sum_j v_{ij} \psi_{ij} \geq \sum_i \sum_j v_{ij}^* \psi_{ij}^* \equiv F^*$$

Let $\psi_{ik}^* = \underset{j}{\text{Min}} \ \psi_{ij}^*$, then

$$F^* = \sum_i (v_{ik}^* \psi_{ik}^* + \sum_{j \neq k} v_{ij}^* \psi_{ij}^*)$$

$$= \sum_i [(V_i - \sum_{j \neq k} v_{ij}^*) \psi_{ik}^* + \sum_{j \neq k} v_{ij}^* \psi_{ij}^*]$$

$$= \sum_i V_i \psi_{ik}^* + \sum_i \sum_{j \neq k} v_{ij}^* (\psi_{ij}^* - \psi_{ik}^*)$$

but $v_{ij}^* \geq 0 \ \forall_i \ \forall_j$ and $\psi_{ij}^* - \psi_{ik}^* \geq 0$ by definition hence

$$F^* \geq \sum_i V_i \psi^*_{ik}$$

i.e. one does no worse by sending all of V_i to the k-th sink which is by definition the costwise nearest.

On the other hand if there are capacity constraints it may happen that the quantity

$$\sum_{\{i \mid \psi^*_{ik} = \psi^*_{i\ell}\}} V_i \text{ is either } > U_\ell \text{ or } < L_\ell$$

in which case the capacity constraints are not obeyed.

Thus, we may let $\alpha_{ij} = v_{ij}/V_i$ and assuming

(A2) There are no capacity constraints at the sinks.

We rewrite (2.1) as

$$\text{Minimize} \quad \sum_{i=1}^{i=n} \sum_{j=1}^{j=m} \alpha_{ij} \psi_{ij}$$

$$\text{S.T.} \sum_{j=1}^{j=m} \alpha_{ij} = 1 \tag{2.2}$$

$$\alpha_{ij} = 0 \text{ or } 1$$

Alternatively, we may take the same course that was taken in converting (1.1) to (1.3), i.e.

Let $M > 0$ be such that $T_i = MV_i$ is an integer and define $N = \sum_i T_i$. Then we may replicate each source an integral number of times T_i and assume

that the commodity of flow comes in indivisible units, say truckloads, i.e.

$$\text{Min} \sum_{i=1}^{i=N} \sum_{j=1}^{j=m} \alpha_{ij} \psi_{ij}$$

$$\text{S.T.} \sum_{j=1}^{j=m} \alpha_{ij} = 1 \tag{2.3}$$

$\alpha_{ij} = 1$ if the i-th source sends its truckload to the j-th sink

$= 0$ otherwise.

In this scheme, then, one may if one desires reintroduce capacity constraints without feeling that one is trampling on a theorem. We take this latter course, but we shall hold off on capacity constraints until Section C below.

It remains only to say something more specific about the functions $\psi(a_i, b_i, x_j, y_j)$. Let us then ask what common sense dictates for the properties of these functions.

First, in any realistic situation we would expect that $\psi_{ij} \geq 0$. (No subsidized routes.)

Second, although there are economic situations in which the costs are not the same in both directions, we are concerned with the movement of refuse in the direction of the disposal sites hence we accept the notion that $\psi(a_i, b_i, x_j, y_j) = \psi(x_j, y_j, a_i, b_i)$ as a useful simplification.

Third, we would expect that when any sink moves close to a source that $\psi_{ij} \to 0$.

Lastly, given a trip origin and destination, common sense tells us that it should cost more to make the trip by way of a detour off the direct path than by way of the direct path.

There is a class of functions having these sensible properties and these are the metrics or distance measuring formulae. One such metric is the Euclidean metric which we have seen in **Chapter** 1:

$$d_E(r, r') = [(x - x')^2 + (y - y')^2]^{1/2} \tag{2.4A}$$

where

$$r = (x, y) \ , \ r' = (x', y')$$

Thus, we shall assume

(A3) The cost is proportional to distance, but the metric we shall use shall be the ℓ_1 , or rectangular distance metric

$$d_{\ell_1}(r, r') = |x - x'| + |y - y'| \tag{2.4B}$$

The name ℓ_1 comes from the name ℓ_p-metrics which are a family of metrics

$$d_{\ell_p}(r, r') = [|x - x'|^p + |y - y'|^p]^{1/p} \tag{2.5}$$

$p > 0$ is an integer

of which the Euclidean metric can be recognized as the second member.

There are several reasons that justify the choice of the ℓ_1-metric. The direct paths of the ℓ_1-metric are in general L-shaped and conform to the experience of pedestrians and traffic in most North American cities; the streets are laid out in a rectangular grid and to go from hither to thither one must follow an L-shaped path up the avenues and down the streets.

Even in cities and metropolitan areas where there are many diagonal streets (e.g. Washington, D.C., Indianapolis, Indiana) it is precisely these streets which one desires to keep free of truck traffic since they are generally crowded with other kinds of traffic.

There are, moreover, many cities which having irregular street patterns are nevertheless polarized as regards speed of travel. Thus, for example, north-south traffic moves at a faster pace in Manhattan Island than does east-west traffic. In such situations the problem can be posed in time proportional co-ordinates.

Finally, as we shall see, we can often obtain the correct E-metric allocation for Cooper's problems by using the ℓ_1-metric, i.e. the ℓ_1-metric provides a useful approximation to the solution of E-metric problems.

Summing up these we may state our problem thus:

Given points called sources located at (a_i, b_i) $i = 1, 2, \ldots, n$. These sources are the points where refuse loads will depart for disposal sites. There are T_i trucks at each source. If we are concerned with capacity constraints at the sinks we define $N = \sum_i T_i$; otherwise $N = n$. We must determine the locations (x_j, y_j) $j = 1, 2, \ldots, m$ and the allocation of sources to sinks on an indivisible basis to

$$\text{Minimize} \quad \sum_{i=1}^{i=N} \sum_{j=1}^{j=m} \alpha_{ij}(|x_j - a_i| + |y_j - b_i|)$$

$$\text{S.T.} \sum_{j=1}^{j=m} \alpha_{ij} = 1 \quad \text{and} \quad \alpha_{ij} = 0 \text{ or } 1 . \qquad (2.6)$$

Section B

Solution of a One Sink Problem

We begin our investigation of the solution of (2.6) by considering the case $n = 1$. Under this assumption, the allocation problem disappears and we have

$$\text{Min} \sum_{i=1}^{i=N} (|x - a_i| + |y - b_i|) \tag{2.7}$$

We note that there are no constraints that relate x and y hence (2.7) can be written as two separate minimization problems

$$\text{Min} \sum_{i=1}^{i=N} |x - a_i|$$

and (2.8)

$$\text{Min} \sum_{i=1}^{i=N} |y - b_i|$$

We concentrate, therefore, on only one of these.

The solution of (2.8) is well known, the answer is given by

Theorem 2: $x = \text{Median}\ \{a_i\}$

$$\begin{aligned} &\text{i.e. for } N \text{ odd}, \quad x = a'_{(N+1)/2} \\ &\text{for } N \text{ even}, \quad a'_{N/2} \le x \le a'_{N/2+1} \end{aligned} \tag{2.9}$$

where $a'_1 \le a'_2 \le \cdots \le a'_N$ and $\{a'_i\} = \{a_i\}$.

Some authors define the median in terms of the minimization property of (2.8), but we shall for the sake of completeness prove the result here.

Proof:

$$f(x) = \sum_{i=1}^{i=N} |x - a'_i| = kx - \sum_{i=1}^{i=k} a'_i - (N - k)x + \sum_{i=k+1}^{i=N} a'_i$$

$$= (2k - N)x - \sum_{i=1}^{i=k} a'_i + \sum_{i=k+1}^{i=N} a'_i$$

where $a'_1 \le a'_2 \le \cdots \le a'_k \le x \le a'_{k+1} \le \cdots \le a'_N$.

Thus, $f(x)$ is a linear function of x within each closed interval $[a_k, a_{k+1}]$ of slope $2k - N$ and is continuous for all x.

Hence, for $k < \frac{N}{2}$, the slope is negative and

for $k \geq \frac{N}{2}$, the slope is non-negative.

The minimum of $f(x)$ occurs, therefore, at $x = a'_{(N+1)/2}$ for N odd and at any point in $[a'_{N/2}, a'_{N/2+1}]$ for N even (slope is zero in this closed interval).

Thus, the pure location problem (2.7) which is the ℓ_1-equivalent of the pure location problem (1.3) has the closed-form solution:

$$x = \text{Median}\,\{a_i\}\ ,\ y = \text{Median}\,\{b_i\} \qquad (2.10)$$

and for the multi-sink problems it follows that

> Given an allocation, the optimal locations for that allocation is gotten by using (2.10) m times where each median is to be taken over the appropriate group of sinkmates.

For brevity we shall refer to this as the Median Principle in the sequel.

Section C

Area Exclusions and the One Sink Problem

It may happen that the solution given by (2.10) allows the minimal point (x, y) to be located in an area which other considerations exclude from eligibility. This means that a constraint must be added to (2.7) expressing the area exclusion. We illustrate with an example (Figure 1-1). The median solution to the six-source, single-sink location problem illustrated is any point on the heavy line AB. Suppose, however, that we are now told that no sinks are allowed to the right of the oblique line PQ. Our intuition suggests that the best location for the sink under this area exclusion is that point in permitted territory which is ℓ_1-closest to the line AB. This is the point C which lies on the ℓ_1-circle (CDEF) of smallest radius (AC) with center (A) on the line AB.

Since intuition is a better source of inspiration than truth, we need a constructive way of solving the problem with area exclusion. To that end, we first state the problem and then convert it to a form amenable to standard methods of solution. To add some generality, we may imagine that there are K straight lines partici-

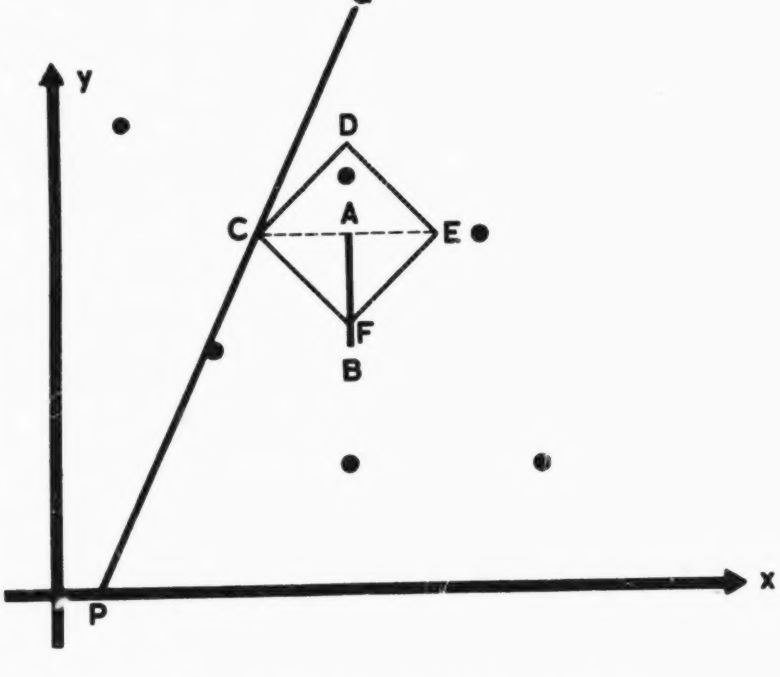

Q
y
D
A
C
E
F
B
P
x

Figure 1-1 Median Solution and an Area Exclusion

pating in the area exclusion so that the point (x, y) must

$$\text{Minimize} \quad \sum_{i=1}^{i=N} (|x - a_i| + |y - b_i|)$$

$$\text{S.T. } c_k x + d_k y \leq e_k \qquad (2.11)$$

$$k = 1, 2, \ldots, K$$

$$\text{and} \quad x, y \geq 0$$

The addition of the non-negativity constraints $x \geq 0$, $y \geq 0$ will be explained shortly. We now introduce the concept of the positive and negative parts of a function, which is

> Suppose we have a function $f(x)$ which may assume any real value $(-\infty < f(x) < \infty)$. Then the positive and negative parts of $f(x)$ are $f^+(x)$ and $f^-(x)$ respectively and are defined as

$$f^{+}(x) = \begin{Bmatrix} f(x) \\ 0 \end{Bmatrix} \quad \text{as} \quad f(x) \begin{Bmatrix} \geq \\ < \end{Bmatrix} 0$$

$$f^{-}(x) = \begin{Bmatrix} 0 \\ -f(x) \end{Bmatrix} \quad \text{as} \quad f(x) \begin{Bmatrix} \geq \\ < \end{Bmatrix} 0 \tag{2.12}$$

From this we may note that

$$\begin{aligned} &\text{a)} \qquad f^{+}(x) - f^{-}(x) = f(x) \\ &\text{b)} \qquad f^{+}(x) + f^{-}(x) = |f(x)| \end{aligned} \tag{2.13}$$

Using this concept we define variables (u_i^{+}, u_i^{-}) and (v_i^{+}, v_i^{-}) as the positive and negative parts respectively of the functions $x - a_i$ and $y - b_i$ respectively. Thus, (2.11) becomes

$$\text{Minimize} \qquad \sum_{i=1}^{i=N} (u_i^{+} + u_i^{-} + v_i^{+} + v_i^{-})$$

$$\begin{aligned} \text{S.T.} \quad & \left.\begin{aligned} u_i^{+} - u_i^{-} &= x - a_i \\ v_i^{+} - v_i^{-} &= y - b_i \end{aligned}\right\} \; i = 1, 2, \ldots, N \\ & c_k x + d_k y \leq e_k \qquad k = 1, 2, \ldots, K \\ & \text{all variables } \geq 0 \end{aligned} \tag{2.14}$$

Thus we have removed the absolute value brackets from the problem and replaced (2.11) by the minimization of a linear function subject to linear constraints, i.e. (2.14) is a linear programming problem and as such can be solved by the simplex algorithm (see [C 6] for example). This algorithm produces non-negative answers only and this is why we added the constraints $x, y \geq 0$ to (2.11). This is a situation which can always be arranged by shifting the origin sufficiently far to the southwest.

There is another case of area exclusion of interest to planners. Suppose there is a simply connected (i.e., all in one piece) zone called the central business district (CBD) whose boundaries are parallel to the axes, and we are forbidden to place a disposal site therein. (See Figure 1-2). Then the area exclusion constraints take the form

$$(x \leq A_1) \ \underline{\text{or}} \ (A_2 \leq x) \ \underline{\text{or}} \ (A_1 \leq x \leq A_2 \ \text{ and } \ y \leq B_1)$$

$$\underline{\text{or}} \ (A_1 \leq x \leq A_2 \ \text{ and } \ y \geq B_2) \qquad (2.15)$$

This is an "exclusive-or" not a "logical-or" so that there is no territory, for example, that simultane-

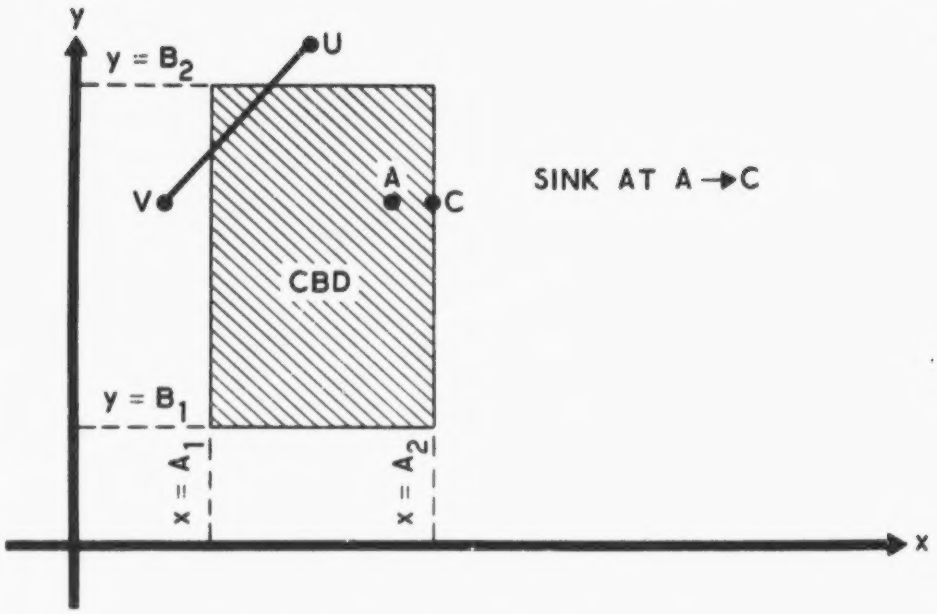

y
y = B_2
U
V
A
C
SINK AT A → C
CBD
y = B_1
x = A_1
x = A_2
x

Figure 1-2 Area Exclusion of a Central Business District

ously obeys $x \leq A_1$ and $x \geq A_2$. This area exclusion literally puts a hole in the solution space and renders the problem "non-convex". This means that the set of eligible points (those outside the CBD) are not a convex set in that every straight line connecting two points in the set is not wholly inside the set; line UV is an example. This is a property that must be possessed by the sets of eligible points (solution space) in order to be solvable by linear programming.

This non-convexity can be repaired by resorting to indicator variables, i.e. variables taking the values 0 or 1 only. We **have** met this type of variable in (2.6) in the α_{ij}'s which expressed the indivisibility of a truckload.

Thus, let M be a very large number (say 100 Max$\{|A_2|, |B_2|\}$) and denote the four disjoint areas described in (2.15) as R_1, R_2, R_3, R_4 respectively. Then we define

$$\left.\begin{aligned} \lambda_k &= 1 \quad \text{if} \quad (x, y) \in R_k \\ &= 0 \quad \text{otherwise} \end{aligned}\right\} k = 1, 2, 3, 4 \qquad (2.16)$$

Then if we define the variables x_k, y_k, $k = 1, 2, 3, 4$ by

$$x = x_1 + x_2 + x_3 + x_4$$
$$y = y_1 + y_2 + y_3 + y_4 \qquad (2.17)$$

the following set of constraints purchases for us the exclusion of the CBD

$$\begin{array}{ll}
\text{a)} & 0 \leq x_1 \leq A_1 \lambda_1 \quad (R_1) \\
 & 0 \leq y_1 \\
\text{b)} & A_2 \lambda_2 \leq x_2 \leq M\lambda_2 \quad (R_2) \\
 & 0 \leq y_2 \\
\text{c)} & A_1 \lambda_3 \leq x_3 \leq A_2 \lambda_3 \quad (R_3) \\
 & 0 \leq y_3 \leq B_1 \lambda_3 \\
\text{d)} & A_1 \lambda_4 \leq x_4 \leq A_2 \lambda_4 \quad (R_4) \\
 & B_2 \lambda_4 \leq y_4 \leq M\lambda_4 \\
\text{e)} & \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \quad \text{(Disjointness)}
\end{array} \qquad (2.18)$$

Summing up we write the CBD model as

$$\text{Min} \sum_{i=1}^{i=N} (u_i^+ + u_i^- + v_i^+ + v_i^-)$$

$$\left.\begin{aligned} u_i^+ - u_i^- &= x_1 + x_2 + x_3 + x_4 - a_i \\ v_i^+ - v_i^- &= y_1 + y_2 + y_3 + y_4 - b_i \end{aligned}\right\} i = 1, 2, \ldots, N \quad (2.19A)$$

(2.18 a - e)

All variables ≥ 0

$\lambda_k = 0$ or 1 only , $k = 1, 2, 3, 4$

This is a mixed-integer programming problem, i.e. it is a linear program some of whose variables may take arbitrary non-negative values while others may take only non-negative integer values, here further specialized to the pair 0 or 1 . A separate listing of appropriate references to works in the area of integer linear programming is included in the bibliography for Part I (Section C).

It may seem fruitless to have thus made contact with the powerful analytical tools of linear and integer linear programming when we return to our original surmise and find that it is true. But we have, in fact, laid the groundwork for multi-sink problems wherein area exclusions may not be trivial.

Some Remarks Concerning the CBD Model.

The CBD model (2.19A) suggests a very natural extension which augments its contact with reality. Suppose we wish to consider the cost of the site which we are seeking to locate and the acquisition cost is different for the regions R_1, R_2, R_3 and R_4. Let these costs be given as A_k, $k = 1, 2, 3, 4$ and let D be a discount factor which gives the transportation segment of total cost the proper scale with respect to the capital expenditure segment, then an altered objective function for (2.19A) is given by

$$\text{Min} \left\{ \sum_{k=1}^{k=4} A_k \lambda_k + D \sum_{i=1}^{i=N} (u_i^+ + u_i^- + v_i^+ + v_i^-) \right\} \qquad (2.19B)$$

This type of problem is termed a "fixed-charge" problem, i.e. the acquisition cost is not increased in proportion to any variable but in full or not at all. We shall encounter these ideas again in Chapter 4.

Section D

Multiple Sink Problems as Integer Programming Problems

Once we advance to two or more sinks to be determined simultaneously, the allocation variables α_{ij}(= 0 or 1) re-enter the problem. We have seen that given an allocation our problem becomes m (the number of sinks) separate location problems which can be solved by the methods of sections B and C . On the other hand, given the locations of the sinks, the allocation is clear: each source to the nearest sink. The complexity of location-allocation models is in the required simultaneity of determination.

Applying the principles of (2.12) and (2.13) to (2.6) we have

$$\text{Minimize} \sum_{i=1}^{i=N} \sum_{j=1}^{j=m} \alpha_{ij}(u^+_{ij} + u^-_{ij} + v^+_{ij} + v^-_{ij})$$

$$\begin{aligned} \text{S.T. } & \left.\begin{aligned} u^+_{ij} - u^-_{ij} &= x_j - a_i \\ v^+_{ij} - v^-_{ij} &= y_j - b_i \end{aligned}\right\} \begin{aligned} i &= 1, 2, \ldots, N \\ j &= 1, 2, \ldots, m \end{aligned} \\ & \sum_{j=1}^{j=m} \alpha_{ij} = 1 \quad i = 1, 2, \ldots, N \\ & \text{all variables } \geq 0 \text{ , } \alpha_{ij} = 0, 1 \text{ .} \end{aligned} \qquad (2.20)$$

This is a problem with linear constraints (except for $\alpha_{ij} = 0, 1$) and a non-linear objective function. We show now how to linearize the objective function.

First, we return to absolute value notation, and defining M to be a very large number, say $M \geq \underset{i}{\mathrm{Max}} |a_i| + \underset{i}{\mathrm{Max}} |b_i|$, we introduce auxillary variables t_{ij} as shown:

$$\mathrm{Min} \sum_{i=1}^{i=N} \sum_{j=1}^{j=m} \left(|x_j - a_i \alpha_{ij}| + |y_j - b_i \alpha_{ij}| - t_{ij} \right)$$

$$\text{S.T.} \sum_{j=1}^{j=m} \alpha_{ij} = 1 \quad \alpha_{ij} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \qquad \text{(i)}$$

$$0 \leq t_{ij} \leq M(1 - \alpha_{ij}) \qquad \text{(ii)} \quad (2.21)$$

$$0 \leq t_{ij} \leq |x_j| + |y_j| \equiv x_j + y_j \quad \text{(iii)}$$

all variables non-negative.

The identity in the last constraint is based on the assumption that the non-negativity of all the x_j, y_j has been assured by a suitable placement of the origin.

Note that $\alpha_{ij} = 1 \Rightarrow t_{ij} = 0$ and $x_j + y_j \geq 0$ (redundant) and the corresponding term of the objective function becomes $|x_j - a_i| + |y_j - b_i|$; whereas $\alpha_{ij} = 0 \Rightarrow t_{ij} \leq M$ (redundant) and $t_{ij} \leq x_j + y_j$, but since $x_j + y_j << M$ and since the process of minimization will force t_{ij} in the absence of any other constraints to be as large as possible, we have $t_{ij} = x_j + y_j$ and the corresponding term of the objective function vanishes. Thus at the points $\alpha_{ij} = 0$ or 1 the objective function behaves like that of (2.6).

This process should more accurately be called a "quasilinearization" since it rests on the non-linear requirement $\alpha_{ij} = 0$ or 1 only. Thus, it is important to note that whereas there is a natural interpretation in (2.6) for $0 < \alpha_i < 1$, such values of α_i in (2.21) may produce ludicrous results. For example, the choice $\alpha_{ij} = 1/m \ \forall i \ \forall j$ reduces to the unconstrained minimization of

$$\sum_{i=1}^{i=N} \sum_{j=1}^{j=N} \left(\left| x_j - \frac{a_i}{m} \right| + \left| y_j - \frac{b_i}{m} \right| - x_j - y_j \right)$$

which bears no relation to the physical situation, i.e., $|x_j - a_i/m|$ fights for a central value for x_j while $-x_j$ fights for as large a positive value as possible.

Returning to (2.21) we may remove absolute value brackets in the same manner as before to get

$$\text{Min} \sum_{i=1}^{i=N} \sum_{j=1}^{j=m} (u_{ij}^{+} + u_{ij}^{-} + v_{ij}^{+} + v_{ij}^{-} - t_{ij})$$

(2.21 (i) - (iii))

and $\qquad\qquad\qquad\qquad\qquad\qquad$ (2.22)

$$u_{ij}^{+} - u_{ij}^{-} = x_j - a_i\,\alpha_{ij}$$

$$v_{ij}^{+} - v_{ij}^{-} = y_j - b_i\,\alpha_{ij}$$

with all variables non-negative.

This is a mixed-integer problem with $4mN + N$ constraints in $6mN + 2m$ variables ignoring the slack variables required to render the inequalities equations. This means prodigious problem sizes for even moderate values of m and N (e.g. $m = 3$, $N = 50$ gives an array 650×906) . There are, however, reasons for believing that the use of integer programming techniques may be feasible for solving large location-allocation problems. These reasons are:

(1) Sparsity and regularity of structure. The density of the structure part of the array of (2.22) is $7/((4m + 1)(3N + 1)) \times 100\%$ sparse. The structure is quite regular and the columns or rows could be generated by algorithm one at a time as needed. See Figure 3.

(2) Great recent advances in methods for solving mixed-integer and integer programs (see bibliography).

(3) The solution process may be accelerated (and often be simply a confirmation of optimality) by use of procedures such as those to be discussed in section F for finding good local optima to be used as advanced starts in the mixed-integer programming formulation.

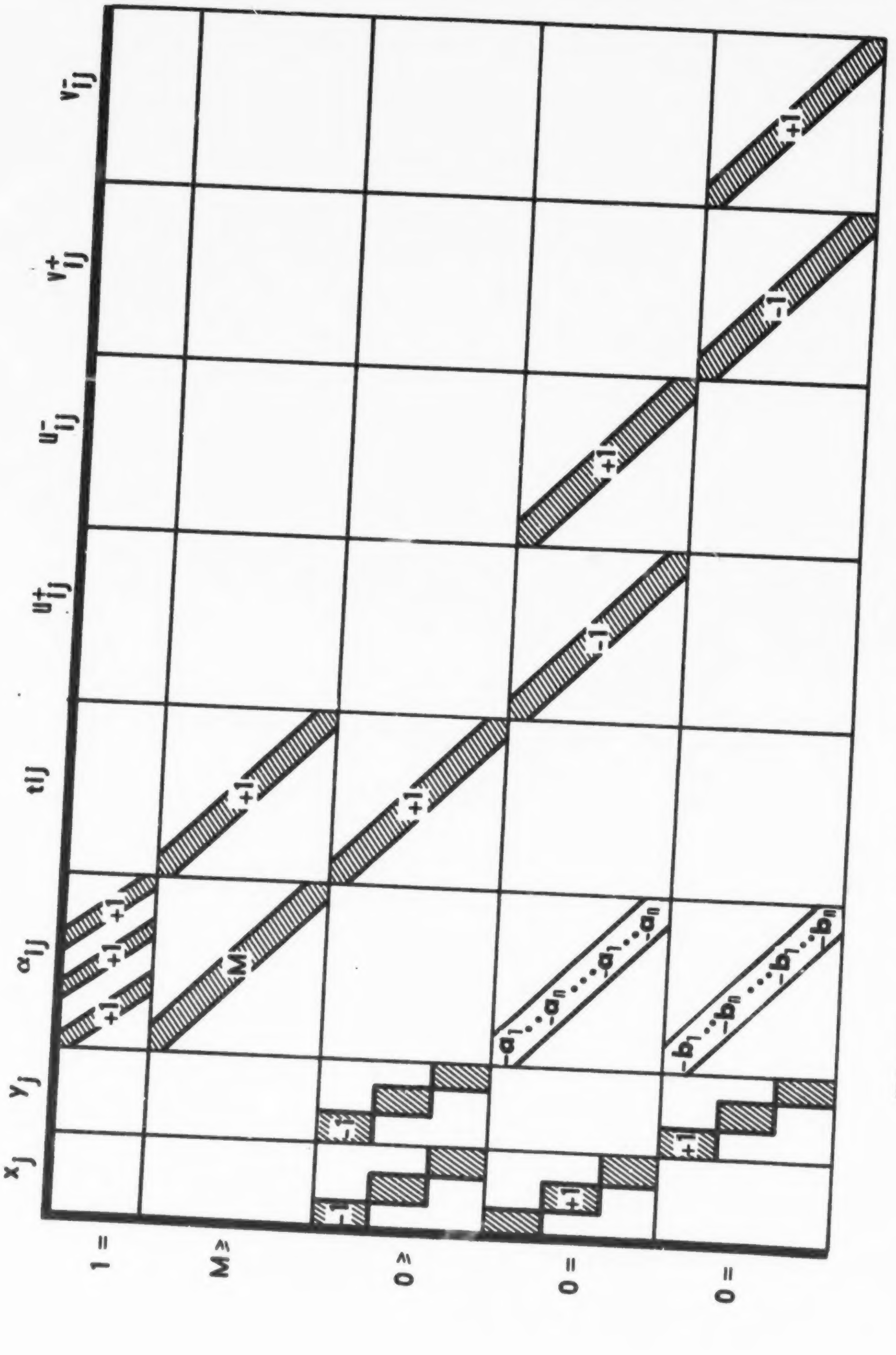

x_j
y_j
α_{ij}
t_{ij}
u^+_{ij}
u^-_{ij}
v^+_{ij}
v^-_{ij}
= 1
≤ M
≤ 0
= 0
= 0
+1
-1
M
$-a_1 \cdots -a_n \cdots -a_1 \cdots -a_n$
$-b_1 \cdots -b_n \cdots -b_1 \cdots -b_n$

Figure 1-3 Structure of Constraint Matrix of (2.22)

Section E

A Theory of Median Sets

In this section, we are going to develop more fully the ideas begun in section B concerning the median nature of the solution of pure ℓ_1-location problems, i.e. the Median Principle. According to this principle, when an allocation is given, the ℓ_1-metric location-allocation problem becomes m separate location problems whereof the solutions are the medians of the m sinkmate groups.

In section F below we are going to develop an algorithm which is like Cooper's Alternate Location and Allocation Algorithm (see page 1-15), i.e. starting with an initial allocation we find points which were better placed in different sinkmate groups, perform the exchange and resolve the location problems. Accordingly, we must know what happens to the location of the median of a sinkmate group when a source is $\left\{\begin{matrix}\text{removed}\\ \text{appended}\end{matrix}\right\}$. To that end, we sharpen our definition of "median" and develop the appropriate theorems.

Definition: Let $\{a_i\}_j$, $\{b_i\}_j$ be the abscissae and ordinates respectively of the sources allocated to the j-th sink. Let

$$A_j = \text{Median } \{a_i\}_j$$

and

$$B_j = \text{Median } \{b_i\}_j$$

according to the definition of median given by (2.9). Note that A_j and B_j are thus either points or closed intervals independently of one another. By the median set M_j of the sources allocated to the j-th sink we shall mean

$$M_j = A_j \times B_j \qquad (2.23)$$

which is short-hand for

M_j = set of all points (x_j, y_j) such that
x_j is taken from the point set A_j and
y_j is taken from the point set B_j .

Thus M_j is either a point, a closed interval parallel to an axis, or a rectangle with sides parallel to the axes.

Theorem 3: Suppose a source is $\left\{\begin{matrix}\text{adjoined to}\\ \text{removed from}\end{matrix}\right\}$ a set of $k\left\{\begin{matrix}\geq 1\\ \geq 2\end{matrix}\right\}$ sources. Let the median sets before and after be denoted by M^k and $\left\{\begin{matrix}M^{k+1}\\ M^{k-1}\end{matrix}\right\}$ respectively. Then the intersection $\left\{\begin{matrix}M^k \cap M^{k+1}\\ M^k \cap M^{k-1}\end{matrix}\right\}$ is not empty and the point in M^k ℓ_1-$\left\{\begin{matrix}\text{nearest}\\ \text{furthest from}\end{matrix}\right\}$ the $\left\{\begin{matrix}\text{adjoined}\\ \text{removed}\end{matrix}\right\}$ point will be in this intersection.

Proof: The proof will be given only for the adjoining part of the theorem.

Let
$$M^k = A^k \times B^k$$

$$M^{k+1} = A^{k+1} \times B^{k+1}$$

where A , B are the one-dimensional median sets of the definition (2.23). Further, let the co-ordinates of the adjoined point be (a', b') and assume $a_1 \leq a_2 \leq \cdots \leq a_k$; $b_1 \leq b_2 \leq \cdots \leq b_k$.

Case 1: k is odd $\Rightarrow A^k = a_{\frac{k+1}{2}}$

(i) $a' < a_{\frac{k-1}{2}} \Rightarrow A^{k+1} = [a_{\frac{k-1}{2}}, a_{\frac{k+1}{2}}]$

(ii) $a_{\frac{k-1}{2}} \leq a' \leq a_{\frac{k+1}{2}} \Rightarrow A^{k+1} = [a', a_{\frac{k+1}{2}}]$

(iii) $a_{\frac{k+1}{2}} \leq a' \leq a_{\frac{k+3}{2}} \Rightarrow A^{k+1} = [a_{\frac{k+1}{2}}, a']$

(iv) $a_{\frac{k+3}{2}} < a' \Rightarrow A^{k+1} = [a_{\frac{k+1}{2}}, a_{\frac{k+3}{2}}]$.

Thus, $A^k \cap A^{k+1} = a_{\frac{k+1}{2}}$ and the closeness condition holds in the x-direction.

Case 2: k is even $\Rightarrow A^k = [a_{k/2}, a_{k/2+1}]$

(i) $a' < a_{k/2} \Rightarrow A^{k+1} = a_{k/2} \Rightarrow A^k \cap A^{k+1} = a_{k/2}$

(ii) $a_{k/2} \leq a' \leq a_{k/2+1}$

$\Rightarrow A^{k+1} = a' \Rightarrow A^k \cap A^{k+1} = a'$

(iii) $a_{k/2+1} < a' \Rightarrow A^{k+1} = a_{k/2+1} \Rightarrow$

$$A^k \cap A^{k+1} = a_{k/2+1} .$$

Thus $A^k \cap A^{k+1}$ is not empty and the closeness condition holds in the x-direction.

Similarly, $B^k \cap B^{k+1}$ is not empty and the closeness condition holds in the y-direction (k even or odd).

Now $(A^k \cap A^{k+1}) \times (B^k \cap B^{k+1}) = (A^k \times B^k) \cap (A^{k+1} \times B^{k+1})$. (This is a self-evident proposition from set theory which will not be proved here.) But since the Cartesian product of two non-empty sets cannot be empty we have

$$M^k \cap M^{k+1} \neq \emptyset .$$

Moreover, the distance from the adjoined point (a', b') to $M^k \cap M^{k+1}$ is the sum of distances from a' to $A^k \cap A^{k+1}$ and from b' to $B^k \cap B^{k+1}$. These component distances have been shown to have the closeness property, hence their sum has it likewise.

We proceed now to the main prop of our algorithm. To facilitate its presentation we adopt the following notation:

p_i : a source located at (a_i, b_i) .

I_j : $\{i | \alpha_{ij} = 1\}$ i.e. the j-th sinkmate group.

M_j : Median Set of $\{p_i | i \in I_j\}$.

S_j : A sink located at $(x_j, y_j) \in M_j$.

$d(p_i, S_j) = |x_j - a_i| + |y_j - b_i|$.

In terms of this notation, the Median Principle stated above becomes

$$\operatorname*{Min}_X \sum_{i \in I_j} d(p_i, X) = \sum_{i \in I_j} d(p_i, S_j) \qquad (2.24)$$

where $X = (x, y)$.

Theorem 4: Let $\{I_j\}$, $\{S_j | S_j \in M_j\}$ be given for $j = 1, 2, \ldots, m$ and for any i let

$$\begin{aligned} \overline{d}_{ij} &= \operatorname*{Max}_{S_j} d(p_i, S_j) \\ \underline{d}_{ij} &= \operatorname*{Min}_{S_j} d(p_i, S_j) \ . \end{aligned} \tag{2.25}$$

Then a necessary and sufficient condition that the solution $[\{I_j \ , \ S_j\}]$ be a local optimum obtained by *the exchange of single sources* is that for all pairs

$$\{(r, q) \mid r \neq q \ , \ 1 \leq r \ , \ q \leq m \ , \ M_r \cap M_q = \emptyset\}$$

$$\overline{d}_{ir} \leq \underline{d}_{iq} \quad \text{for each} \quad i \in I_r \ .$$

That is, the value of the objective function can decrease if and only if the largest distance from a source p_i to its own median set (M_r) exceeds the smallest distance to a point in any other median set (M_q) .

Proof: Assume the condition of the theorem holds, but the solution is not a local optimum obtainable in the manner indicated. Then there exists a source p_h , $h \in I_r$ such that if p_h is allocated to some other sink $S_q \in M_q$, the value of the function will decrease.

Let $\quad I_r' = I_r - h = I_r \cap C(h)$ (complimentation)

$\quad I_q' = I_q \cup h$

and let $\bar{S}_{hr} \in M_r$ be such that $d(p_h, \bar{S}_{hr}) = \bar{d}_{hr}$

$\quad \underline{S}_{hq} \in M_q$ be such that $d(p_h, \underline{S}_{hq}) = \underline{d}_{hq}$.

Further, let F , F' be the values of the objective function before and after the re-allocation of p_h to S_q . Then

$$\sum_{j=1}^{j=m} \sum_{i \in I_j} d(p_i, S_j) \equiv w + \sum_{i \in I_r} d(p_i, S_r) + \sum_{i \in I_q} d(p_i, S_q)$$

$$= w + \sum_{i \in I_r'} d(p_i, \bar{S}_{hr}) + \bar{d}_{hr} + \sum_{i \in I_q} d(p_i, \underline{S}_{hq})$$

$$\leq w + \sum_{i \in I_r'} d(p_i, \bar{S}_{hr}) + \underline{d}_{hq} + \sum_{i \in I_q} d(p_i, \underline{S}_{hq})$$

by assumption

$$= w + \sum_{i \in I_r'} d(p_i, \bar{S}_{hr}) + \sum_{i \in I_q'} d(p_i, \underline{S}_{hq}) = F' \, .$$

The question now is whether on shifting sinks from $\overline{S}_{hr} \in M_r$ and $\underline{S}_{hq} \in M_q$ to $S_r' \in M_r'$ and $S_q' \in M_q'$ respectively, we will by the Median Principle reduce F' below F. The answer is no, because by theorem 3 $\overline{S}_{hr} \in M_r' \cap M_r$ and $\underline{S}_{hq} \in M_q \cap M_q'$. Hence, F is optimal (locally) and the condition is sufficient.

On the other hand, if the condition does not hold for some source p_h, $h \in I_r$, then p_h may by theorem 3 be re-allocated to $\underline{S}_{hq}$ for a net decrease in F of $\overline{d}_{hr} - \underline{d}_{hq}$. Hence, the original value of F was not optimal (locally) and the condition is necessary. Q.E.D.

An auxillary device for reducing the value of F is provided by the following result.

Theorem 5: If ℓ_1-paths connecting a pair of distinct sources to their respective, distinct sinks are unavoidably crossed then either

(a) The solution in which this occurs is not optimal, or

(b) If it is optimal, there exists an alternate, uncrossed optimum.

Proof: Suppose for $m \geq 2$ that two sources p_1, p_2 are connected to a pair of sinks S_1 and S_2 respectively and their paths are crossed. (See Figure 4a.)

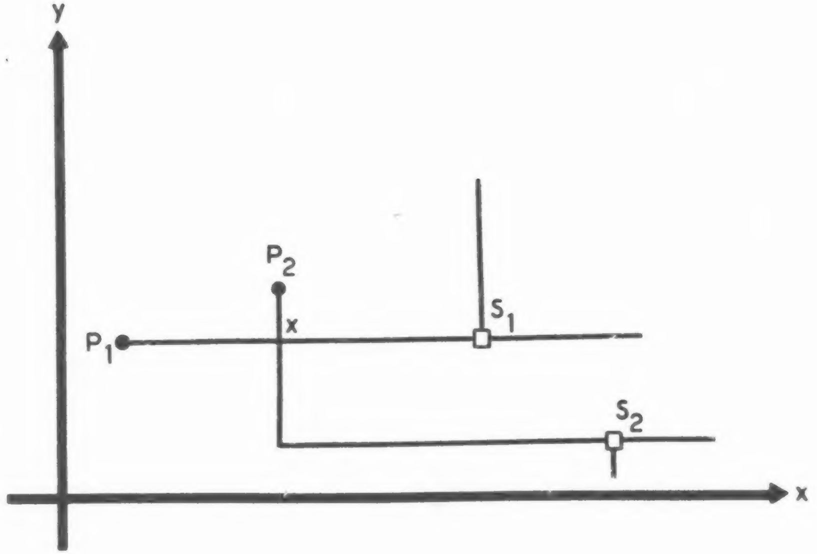

y
P$_1$
P$_2$
x
S$_1$
S$_2$
x

Figure 1-4a Paths from Sources to Sinks Unavoidably Crossed

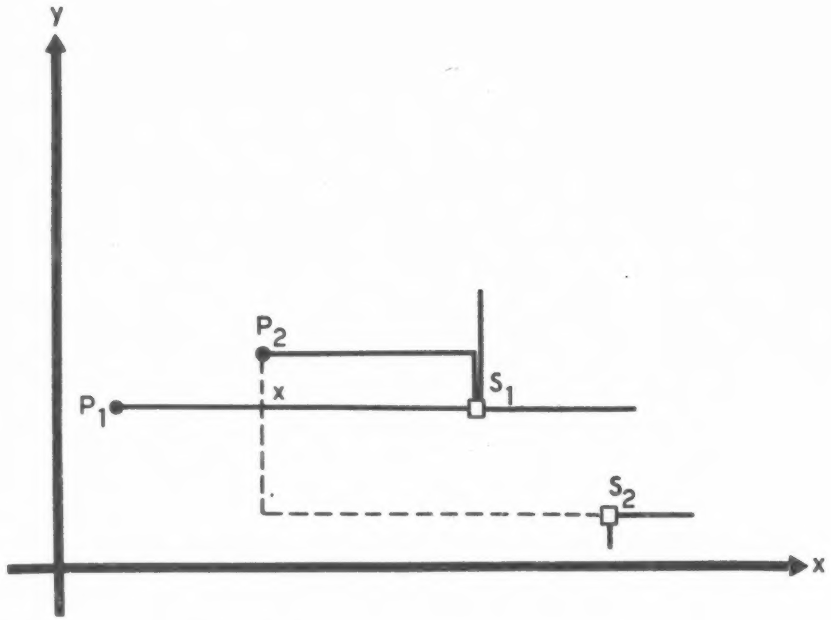

y
P_2
P_1
x
S_1
S_2
x

Figure 1-4b Reallocation to Cause Uncrossing

Let x denote the point of crossing. Then either

$$d(x, S_1) > d(x, S_2) ,$$

$$d(x, S_1) < d(x, S_2) ,$$

or

$$d(x, S_1) = d(x, S_2) .$$

In the first case, re-allocate p_1 to S_2, improving the solution by at least $d(x, S_1) - d(x, S_2)$. In the second case (Figure 4a), re-allocate p_2 to S_1 improving the solution by at least $d(x, S_2) - d(x, S_1)$. This re-allocation is illustrated in Figure 4b.

In both of these cases, an improvement being possible, neither could have been optimal. In the third case, the solution may or may not be optimal. If it was not optimal, either of the above re-allocations can cause at worst an alternate solution with the same value of F which does not have a crossed path. If the solution was optimal, either re-allocation causes an alternate, uncrossed optimum.

It should be noted that many paths may be crossed so that the process given above may have to be repeated.

Moreover, the re-allocation may cause paths previously uncrossed to become crossed. Therefore, the term "uncrossed" as used in the preceding discussion should be taken to refer only to the given crossing. It is clear, however, that iterated use of the above procedure will finally lead to a totally uncrossed solution, optimal or not. Q.E.D.

In view of this theorem, it is reasonable to require that any starting procedure for an algorithm produce uncrossed starting solutions.

Section F

An Approximating Algorithm Based on the Theory of Median Sets

As stated above, we have been erecting a foundation for a single point exchange algorithm. We may pause, though, before stating this algorithm to consider the possibility of multi-point exchanges.

When more than one point is removed from or adjoined to a group, the possible movements of the altered median set become quite complicated. For example, $M^k \cap M^{k+1} \neq \emptyset$ and $M^{k+1} \cap M^{k+2} \neq \emptyset$ do not imply that $M^k \cap M^{k+2} \neq \emptyset$. Hence we lose the prop of our theory which depends on knowing where the median sets go.

Another approach to multi-point methods is had via graphical means. Starting with some initial solution we draw about each sink the smallest ℓ_1-circle (a diamond, see Figure 1) that just covers all its sources. The inclusion of alien sources in such circles can thus make profitable exchanges of several points quite obvious, visually. The difficulty with this method is that when the median set is not a unique point, the arbitrary selection of a point as the center of the circle may be the wrong choice. As a practical matter, the method has proved in several instances incapable of improving verified local optima.

The single-point exchange algorithm consists of the following steps:

(1) Make an initial allocation of sources to sinkmate groups and determine their median sets. We shall say more about this in Section G below and in Chapter 1.3.D.

(2) Set $r = 1$ (sink number) and $k = 0$ (exchange counter).

(3) For each $i \in I_r$ compare $\bar{d}_{ir}$ vs $\underline{d}_{iq}$, $q \neq r$.

(4) If all $\bar{d}_{ir} \leq \underline{d}_{iq}$ go to step 7; otherwise go to step 5.

(5) For some $h \in I_r$, $\bar{d}_{hr} > \underline{d}_{hq}$: move S_r to $\bar{S}_{hr}$, S_q to $\underline{S}_{hq}$ and re-allocate h to I_q. Increase k by 1.

(6) Go to step 3.

(7) If $r \neq m$, increase r by 1 and return to step 3.

If $r = m$ and $k \neq 0$, return to step 2.

If $r = m$ and $k = 0$, the algorithm ends with a local optimum determined.

This version of the algorithm removes all "better elsewhere" points from a sinkmate group before proceeding to the next group. Thus, it is not inconceivable that a source may switch back and forth several times before settling down.

Another approach to an algorithm would be to go through all points and choose that exchange for which $\bar{d}_{hr} - \underline{d}_{hq}$ is largest. Furthermore, for $m \geq 4$ two or more such exchanges might be sought. While such measures *may* in general lead to fewer solution iterations, there is no guarantee that the solution it reaches will have any lower a value of F than the algorithm stated.

There are two additional comments apropos at this point. First, the algorithm as presently constituted, has nothing to say about ties, i.e., when some $\bar{d}_{hr} = \underline{d}_{hq}$ and no $\bar{d}_{hr} > \underline{d}_{hq}$. Second, it is clear that any sinkmate group which consists of a single point (a singleton) may be augmented by transfer from other sinkmate groups, but will never be put into another group. This means that an initial solution should probably try to avoid singletons.

To finish this discussion of the algorithm, we illustrate it by an example taken from Cooper [A 5], who gives eight such examples each having $n = 7$ and $m = 2$. The example used here is the first of these. The results on all eight examples are compared with Cooper's E-metric results in Table 2.

The source locations are shown in Table 1-1. We take our initial guess at an allocation to be

$$I_1 = \{1, 2, 3, 4\}$$

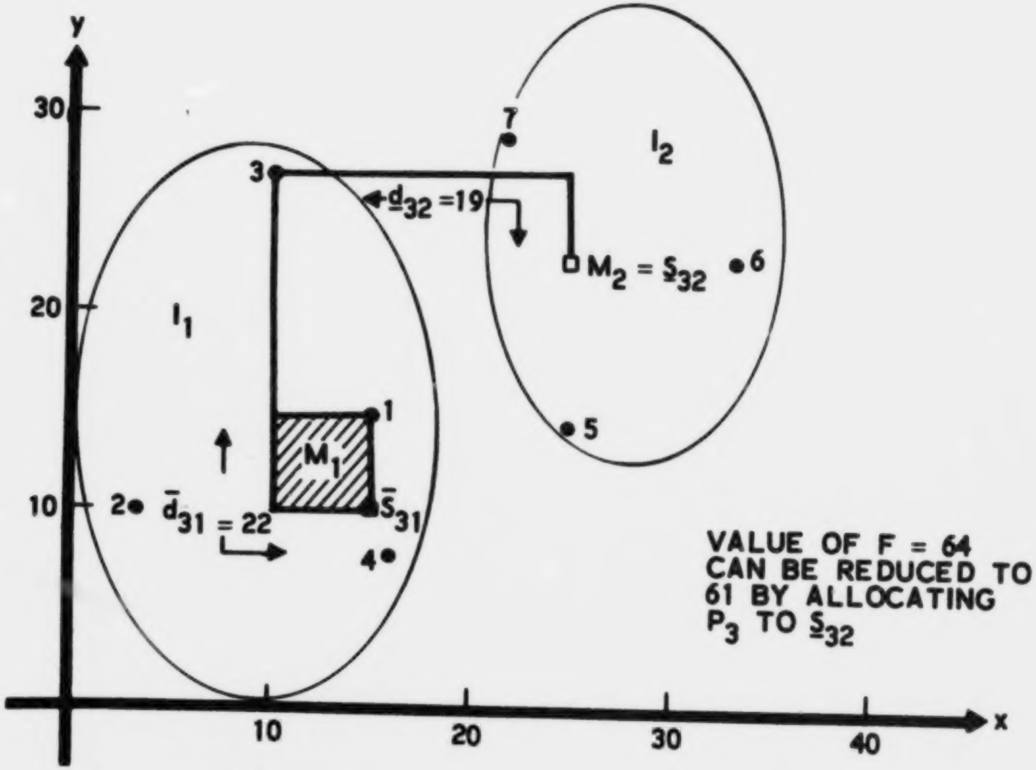
$$I_2 = \{5, 6, 7\}$$

as illustrated in Figure 1-5. Figures 1-6 and 1-7 show the balance of the solution with the computations required appearing therein.

Further investigation shows that the solution of Figure 1-7 is actually the global optimum. That the algorithm can stall at non-global local optima is apparent from Table 1-2. Despite the possibility, Table 1-2 shows that the alternating location-allocation or single point exchange algorithm is an excellent means for determining an E-metric allocation. Moreover, we observe that $F_{\ell_1} \geq F_E$. This is a good upper bound on F_E but the result cannot be proved, i.e. there may be problems where $F_{\ell_1} < F_E$ since the optimal ℓ_1 and optimal E allocations are not necessarily identical.

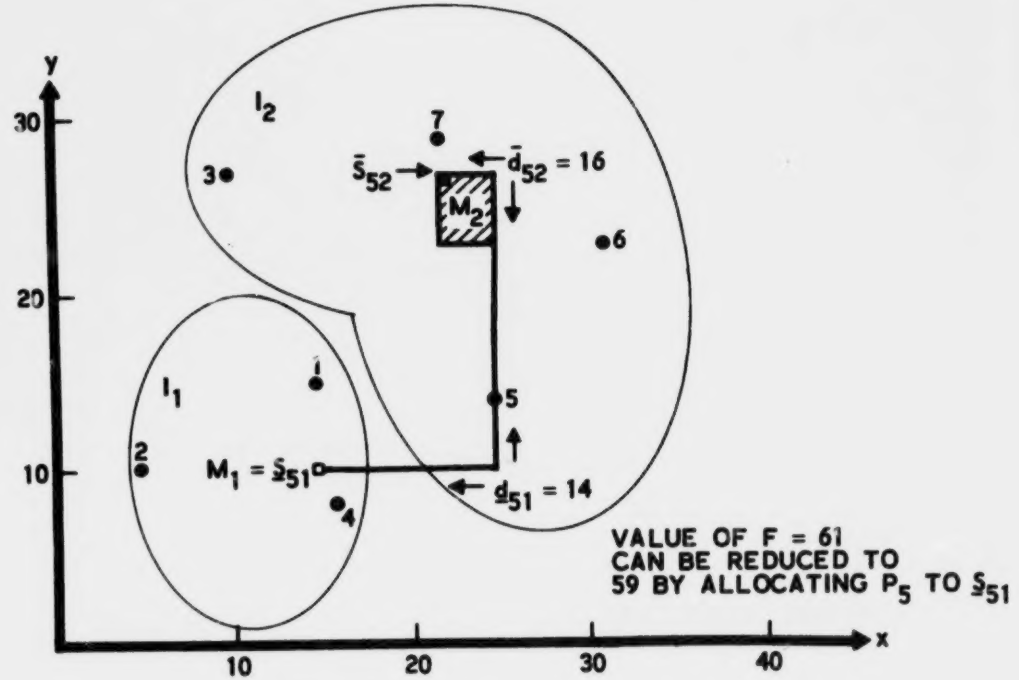
i	a_i	b_i
1	15	15
2	5	10
3	10	27
4	16	8
5	25	14
6	31	23
7	22	29

Table 1-1 Source Locations for Example 1


y
30
20
10
x
10
20
30
40
I_1
I_2
1
2
3
4
5
6
7
M_1
$\bar{S}_{31}$
$\bar{d}_{31} = 22$
$\underline{d}_{32} = 19$
$M_2 = \underline{S}_{32}$

VALUE OF F = 64
CAN BE REDUCED TO
61 BY ALLOCATING
P_3 TO $\underline{S}_{32}$

Figure 1-5 Initial Solution for Example 1


y
30
20
10
x
10
20
30
40
I_2
I_1
3
7
$\bar{s}_{52}$
$\bar{d}_{52} = 16$
M_2
6
1
5
2
$M_1 = \underline{s}_{51}$
4
$\underline{d}_{51} = 14$

VALUE OF F = 61
CAN BE REDUCED TO
59 BY ALLOCATING P_5 TO $\underline{s}_{51}$

Figure 1-6 Result of First Exchange in Example 1

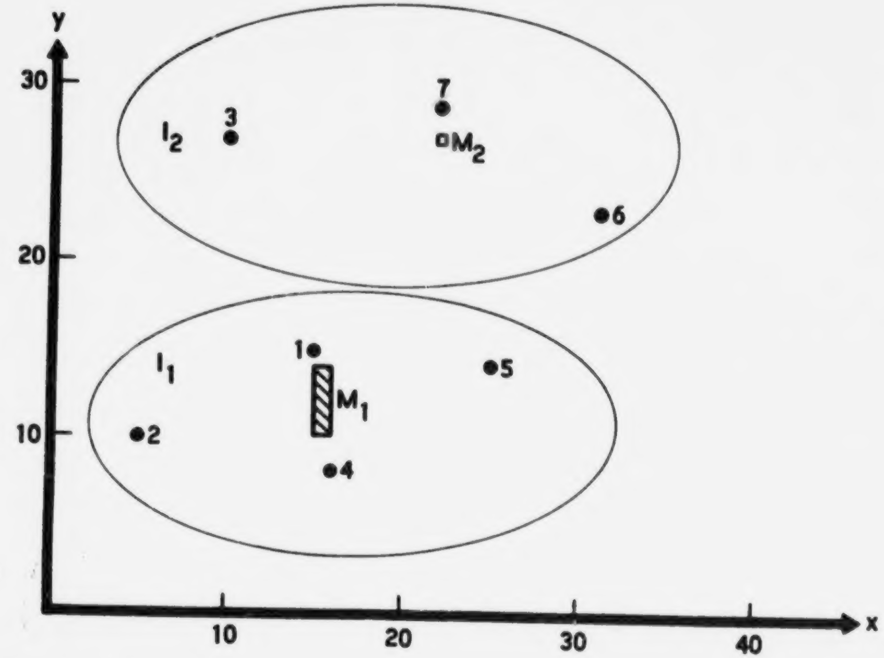


Figure 1-7 Result of Second Exchange in Example 1, Local Optimum Solution

CASE 1	1 2 3 4 5 6 7
SOURCE COORDINATES: ALLOCATIONS: ℓ_1 - ALTERNATE ALLOCATIONS: VALUE OF OBJ FUNCTION:	(15,15) (5,10) (10,27) (16,8) (25,14) (31,23) (22,29) E: (1,2,4,5), (3,6,7); ℓ_1: (1,2,4,5), (3,6,7) E: 50.450 ± 0.001 ; ℓ_1: 59
CASE 2	1 2 3 4 5 6 7
SOURCE COORDINATES: ALLOCATIONS: ℓ_1 - ALTERNATE ALLOCATIONS VALUE OF OBJ FUNCTION:	(6,8) (6,32) (20,8) (20,20) (20,32) (36,8) (36,32) E: (1,3,4,6), (2,5,7); ℓ_1: (1,3,4,6), (2,5,7) (1,3,6), (2,4,5,7) E: 72.002 ± 0.001 (?); ℓ_1: 72
CASE 3	1 2 3 4 5 6 7
SOURCE COORDINATES: ALLOCATIONS: ℓ_1 - ALTERNATE ALLOCATIONS: VALUE OF OBJ FUNCTION:	(8,12) (5,19) (5,26) (5,32) (35,20) (35,26) (35,31) E: (1,2,3,4), (5,6,7); ℓ_1: (1,2,3,4), (5,6,7) E: 38.325 ± 0.001 ; ℓ_1: 41
CASE 4[1]	1 2 3 4 5 6 7
SOURCE COORDINATES: ALLOCATIONS: ℓ_1 - ALTERNATE ALLOCATIONS: VALUE OF OBJ FUNCTION:	(5,23) (9,32) (15,23) (21,32) (26,23) (31,32) (16,12) E: (1,2,3,7), (4,5,6); ℓ_1: (1,3,5,7), (2,4,6) E: 48.850 ± 0.001 ; ℓ_1: 55
CASE 5	1 2 3 4 5 6 7
SOURCE COORDINATES: ALLOCATIONS: ℓ_1 - ALTERNATE ALLOCATIONS: VALUES OF OBJ FUNCTION:	(8,10) (8,26) (11,20) (17,15) (17,22) (24,17) (31,19) E: (1,2,3,5), (4,6,7); ℓ_1: (1,2,3,5), (4,6,7) E: 36.033 ± 0.001 ; ℓ_1: 48
CASE 6	1 2 3 4 5 6 7
SOURCE COORDINATES: ALLOCATIONS: ℓ_1 - ALTERNATE ALLOCATIONS: VALUE OF OBJ FUNCTION:	(6,31) (13,24) (13,31) (20,24) (20,17) (27,17) (27,1) E: (1,2,3,4), (5,6,7); ℓ_1: (1,2,3,4), (5,6,7) (1,2,3), (4,5,6,7) E: 36.175 ± 0.001; ℓ_1: 44
CASE 7[2]	1 2 3 4 5 6 7
SOURCE COORDINATES: ALLOCATIONS: ℓ_1 - ALTERNATE ALLOCATIONS: VALUE OF OBJ FUNCTION:	(2,19) (35,11) (31,33) (25,26) (18,23) (18,16) (11,33) E: (2), (1,3,4,5,6,7); ℓ_1: (3,4,7), (1,2,5,6) E: 59.716 ± 0.001 ; ℓ_1: 74
CASE 8[3]	1 2 3 4 5 6 7
SOURCE COORDINATES: ALLOCATIONS: ℓ_1 - ALTERNATE ALLOCATIONS: VALUE OF OBJ FUNCTION:	(28,6) (28,33) (33,17) (33,23) (39,6) (39,33) (6,10) E: (1,5,7), (2,3,4,6); ℓ_1: (1,5,7), (2,3,4,6) (1,3,5,7), (2,4,6) E: 62.204 ± 0.001 ; ℓ_1: 74

[1] (1,2,3,7),(4,5,6) IS A LOCAL OPTIMUM; OBJ FUNCTION = 56

[2] (2),(1,3,4,5,6,7) IS A LOCAL OPTIMUM: OBJ FUNCTION = 77

[3] (7),(1,2,3,4,5,6) IS A LOCAL OPTIMUM: OBJ FUNCTION = 82

Table 1-2 Comparison of Results ℓ_1 vs. E-metric

Section G

Finding Initial Solutions

We return now to the problem of providing an initial allocation for getting the algorithm started. There is no doubt that the best method for small problems is the use of the human eye and educated guessing. For larger problems there are two methods that are relevant both to the ℓ_1-metric and the E-metric cases. The more powerful of these methods is based on the minimum elemination gain algorithm (MEGA) which is discussed in the next chapter. Hence, we defer discussion of this method until then.

We present here a method for finding an initial solution which is based on the use of the single point exchange algorithm on two simpler problems derived from the original problem. These problems, called the horizontal and the vertical problems, are one-dimensional problems gotten by projecting the sources on the abscissa and the ordinate respectively. If two points project to the same abscissa, they retain their separate identities. Thus, the horizontal problem corresponding to Example 1, Figures 1-5 to 1-7, is shown in 1-8 and the vertical problem in Figure 1-9.

We note that these one-dimensional problems are easily solved because

(a) The tests of the algorithm must be performed on only two points as regards a pair of sinks. These are the "boundary" points, e.g. p_4 and p_7 in Figure 1-8 and p_1 and p_6 in Figure 1-9.

(b) For $m \geq 3$, the test need only be performed on neighboring sinkmate groups.

The optimal horizontal and vertical solutions are illustrated in Figures 1-8 and 1-9.

To make use of these solutions for the two-dimensional problem we construct an $N \times N$ table of coincidences as follows:

If points p_k and p_ℓ appear in the same sinkmate group in the $\left\{\begin{matrix}\text{horizontal}\\\text{vertical}\end{matrix}\right\}$ solution, i.e. are $\left\{\begin{matrix}\text{horizontal}\\\text{vertical}\end{matrix}\right\}$ sinkmates, enter a $\left\{\begin{matrix}h\\v\end{matrix}\right\}$ in the k, ℓ box of the table. The result of this operation is shown in Table 1-3.

We reason now as follows: If a pair of points have both an h and a v in their table entry then they are a favored combination and should appear in the same sinkmate group in the initial solution; If there is

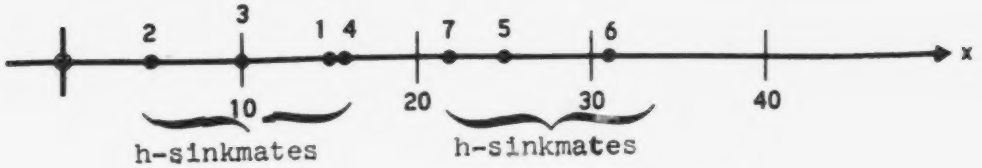

2
3
1 4
7
5
6
x
10
20
30
40
h-sinkmates
h-sinkmates

Figure 1-8 Horizontal Problem Corresponding to Example 1, Figures 1-5 to 1-7

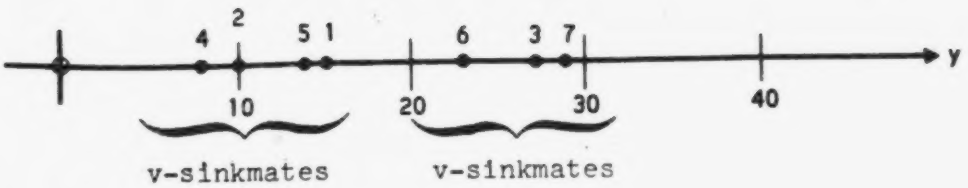

4
2
5 1
6
3 7
y
10
20
30
40
v-sinkmates
v-sinkmates

Figure 1-9 Vertical Problem Corresponding to Example 1, Figures 1-5 to 1-7

neither an h nor a v we shall see that this pair do not become sinkmates; If there is either an h alone or a v alone we shall be arbitrary.

Accordingly, we may interpret Table 1-3 to favor an initial allocation of $\{1, 2, 4\}$, $\{6, 7\}$ with the placement of p_3 and p_5 yet to be decided. Using the arbitrary selection,

$$I_1 = \{1, 2, 3, 4\}$$

$$I_2 = \{5, 6\ 7\}$$

we have the initial solution of Figure 1-5. If we had made the opposite choice $I_1 = \{1, 2, 4, 5\}$, $I_2 = \{3, 6, 7\}$, we would have had the optimal solution of Figure 1-7.

The method presented here will not work if all the sources p_i are scattered about a horizontal or vertical line. If this occurs the horizontal or vertical solution (respectively) should be used as the initial solution.

$\kappa \searrow \ell \rightarrow$	2	3	4	5	6	7
1	hv	h	hv	v		
2		h	hv	v		
3			h		v	v
4				v		
5					h	h
6						hv

Table 1-3 Coincidence Table for Horizontal and Vertical Solutions of Figures 1-8 and 1-9

CHAPTER 3

SELECTION-ALLOCATION MODELS

Section A

Mathematical Definition and Assumptions

In the introduction (section C) and in Chapter 1 we have alluded briefly to the manner in which selection-allocation problems arise from the location-allocation models of Chapter 2. We shall discuss the origin, nature, characterizations and methods of solving these problems more fully in this chapter and the next.

It may be noted that section C of Chapter 2 which dealt with area exclusions and the one sink problem was not succeeded in due course by a section dealing with area exclusions and multiple sink problems. It is clear from section C and D of Chapter 2 that the model (2.22) can be augmented by an appropriate set of area-excluding linear constraints to achieve this end. Further reflection on the real world indicates, however, that area exclusions for aesthetic, political, zoning, or economic reasons are so widespread that the area available for location of sinks is likely to dwindle to a few parcels whose locations are known. On the other hand, Cooper simplifies the solution of location-allocation problems by using the sources themselves as a restricted solution space for the sink locations. Finally, in setting up regional economic plans for product distribution it frequently is desirable to restrict the choice of distribution centers (our sinks) to cities, excluding the countryside (e.g., Maranzana [A 24]).

For these reasons we are brought to consider selection-allocation problems which are stated thusly:

Given: (1) A number n of points called sources, whose locations are known, but are not germane to the problem.

(2) The amount of a homogeneous commodity of flow present at each source.

(3) A number S of points called "eligible sink sites" whose locations are likewise known, but are also not germane to the problem.

(4) A number $1 < m < S$ which is the number of sinks to be selected from the set of eligible sink sites.

(5) $n \cdot S$ numbers d_{ij} representing the cost of sending a unit of the commodity from the i-th source to the j-th sink.

(6) Capacity ceilings and floors $(+\infty, 0$ respectively if need be) at each sink.

To Determine:

(1) The set of sinks to be selected from amongst the set of eligible sink sites.

(2) The amount of commodity sent from each source to each selected sink.

Such that: The total weighted transportation cost is minimized within the capacity constraints (if any).

We may as we have done for location-allocation problems show that in the absence of capacity ceilings (or floors) there is an all-or-nothing solution at least as good as any "continuous" solution (Theorem 1). Thus we list our assumptions:

(1) Each source has an indivisible unit amount to send. We achieve this, if necessary by using the replication process indicated in Chapter 2.

(2) Capacity ceilings or floors do not apply.

(3) The numbers (not functions) d_{ij} are given, are independent of receipts at a sink, may or may not be proportional to distance, and if they are, no choice of metric is implied.

Section B

Characterization As a Network Flow Model

The easiest characterization of a selection-allocation problem is perhaps as a network flow model. To understand this we define the following:

$$
\begin{aligned}
x_{ij} &= 1 && \text{if the i-th source sends its unit to the j-th sink} \\
&= 0 && \text{otherwise} \\
y_j &= && \text{the amount received at the j-th sink.} \\
\alpha_j &= 1 && \text{if the j-th sink is selected (accepted).} \\
&= 0 && \text{otherwise} \\
\rho_j &= 1 && \text{if the j-th sink is rejected} \\
&= 0 && \text{otherwise}
\end{aligned}
\qquad (3.1)
$$

The model consists of two network systems. These are shown in Figures 1-10a and 1-10b. Figure 1-10a represents the "physical" system. Shown on the left are the sources. Each source is connected to every one of the eligible sink sites which are represented as nodes in the center. Each source receives a unit inflow which is divided up into flows x_{ij} to the eligible sinks. The flow

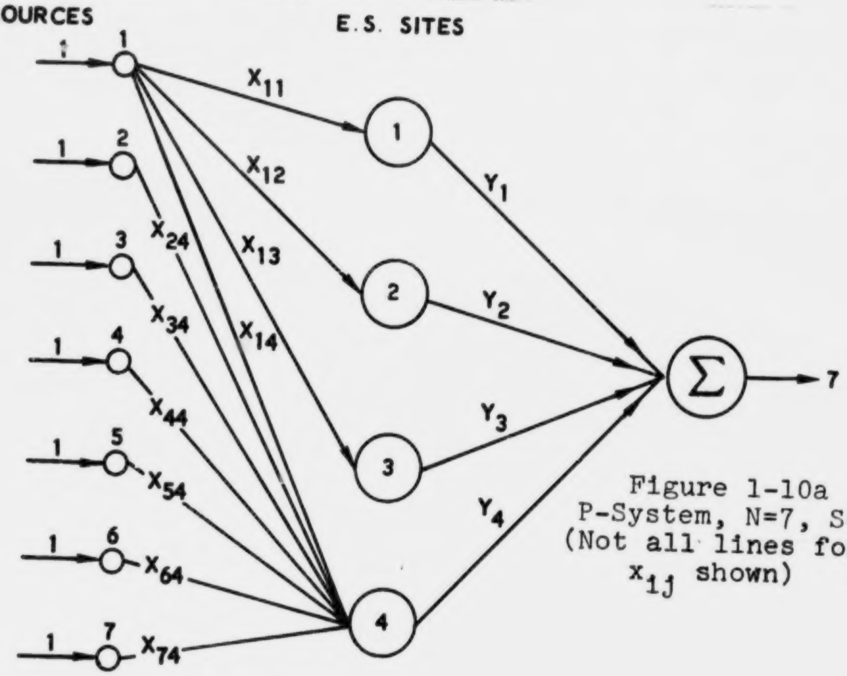

SOURCES
E.S. SITES
1
2
3
4
5
6
7
1
1
1
1
1
1
1
X11
X12
X13
X14
X24
X34
X44
X54
X64
X74
1
2
3
4
Y1
Y2
Y3
Y4
Σ
7

Figure 1-10a
P-System, N=7, S=4
(Not all lines for x_{ij} shown)

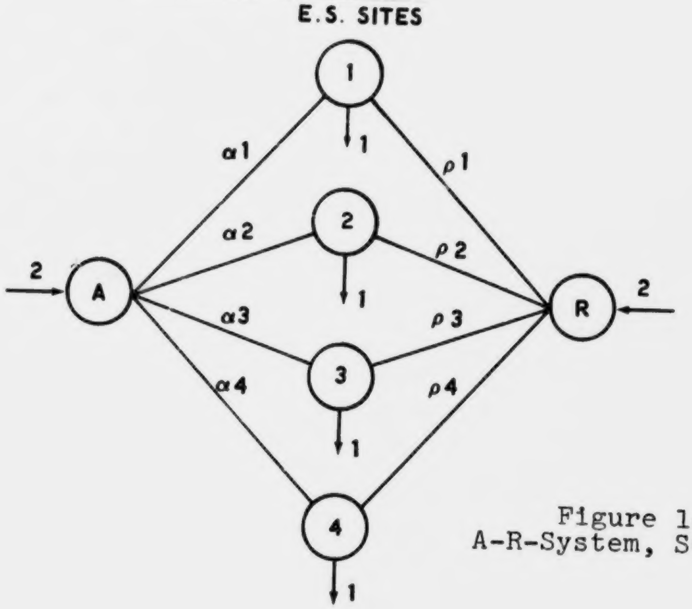

E.S. SITES
1
2
3
4
A
R
2
2
α1
α2
α3
α4
ρ1
ρ2
ρ3
ρ4
1
1
1
1

Figure 1-10b
A-R-System, S=4, m=2

received at each sink is collected as flows y_j which are sent to a figurative summing node (Σ) on the right. The outflow N from the Σ node exactly matches the inflow to the sources.

The equations which represent this network are obtained from the Kirchoff node conditions. These equations are

$$\begin{aligned} &\sum_{j=1}^{j=S} x_{ij} = 1 \qquad \text{(i)} \\ &\sum_{i=1}^{i=N} x_{ij} = y_j \qquad \text{(ii)} \\ &\sum_{j=1}^{j=S} y_j = N \qquad \text{(iii)} \end{aligned} \qquad (3.2)$$

Equation (i) expresses the idea that the total outflow from any source is equal to the total inflow which is 1. Equation (ii) says that the receipts at a sink equals the amount sent to the sink. Equation (iii) says that the amount received at all sinks equals the amount received at all sources.

Note that this physical (P) system says nothing about the notion of selecting some m out of S eligible sink sites and rejecting the other $S - m$. This idea is expressed by the accept-reject (AR) system of Figure 10b. The entities represented here are except for the eligible sink sites contrivances that fit the idea of acceptance-rejection and have no real physical meaning.

Two nodes, the accept (A) node and the reject (R) node, receive quantities m and $S - m$ of "accept" and "reject" substance respectively. The accept node divides its m units of accept-substance into S flows α_j going to the eligible sink sites. Similarly, the reject node divides its $S - m$ units of reject-substance into S flows ρ_j going to the eligible sink sites. At the eligible sink sites the accept and reject flows are received and siphoned out of the system, one unit of something escaping from each eligible sink site. The equations of flow for the A - R system are

$$\begin{aligned} &\sum_{j=1}^{j=S} \alpha_j = m && \text{(i)} \\ &\sum_{j=1}^{j=S} \rho_j = S - m && \text{(ii)} \qquad (3.3) \\ &\alpha_j + \rho_j = 1 && \text{(iii)} \end{aligned}$$

Thus far, there is nothing which links the two systems, yet we realize that in the P-system a receipt y_j cannot be >0 if the corresponding eligible sink site has been rejected ($\alpha_j = 0$, $\rho_j = 1$) in the A-R system. We express this mathematically as

$$0 \leq y_j \leq N\alpha_j \qquad (3.4)$$

These constraints are termed coupling conditions and are the only constraints containing variables from both the P and A-R systems. The total model is then

$$\text{Minimize} \quad \sum_{i=1}^{i=N} \sum_{j=1}^{j=S} d_{ij}\, x_{ij}$$

$$\left.\begin{array}{ll} \text{S.T.} \displaystyle\sum_{j=1}^{j=S} x_{ij} = 1 & i = 1, 2, \ldots, N \\ \displaystyle\sum_{i=1}^{i=N} x_{ij} - y_j = 0 & j = 1, 2, \ldots, S \\ \displaystyle\sum_{j-1}^{j=S} y_j = N & \end{array}\right\} P \quad (3.5)$$

$$\left.\begin{aligned} &\sum_{j=1}^{j=S} \alpha_j = m \\ &\sum_{j=1}^{j=S} \rho_j = S - m \\ &\alpha_j + \rho_j = 1 \quad j = 1, 2, \ldots, S \end{aligned}\right\} \text{A.R.}$$

$$y_j \leq N\alpha_j \quad j = 1, 2, \ldots, S \quad \text{Coupling}$$

$$x_{ij}, \alpha_j, \rho_j = 0 \text{ or } 1 \text{ only}$$

$$y_j \geq 0 \text{ integer.}$$

Several things should be noted about this (all) integer linear programming problem:

(1) As we have noted, both the P and A-R systems are representable as network flows. Therefore, either system alone solves easily and without extra effort with all-integer answers. It is the imposition of the coupling constraints that requires special integer methods on the system as a whole.

(2) The constraint matrix of (3.5) is block angular, i.e., the form is:

$$\begin{array}{|c|c|}\hline P & 0 \\ \hline 0 & A-R \\ \hline \multicolumn{2}{|c|}{C} \\ \hline\end{array}$$

Such problems have been termed "multi-copy" or "multi-page" programs and extensive work has been done on efficient algorithms to capitalize on this structure (see [C 6] for non-integer problems).

(3) The formulation of (3.5) makes it apparent that a selection-allocation problem is equivalent to a transportation problem (see [C 6]) which has the added stipulation that some of the warehouses must be shut down.

(4) It is extremely easy to generate solutions to the problem: pick any m of the eligible sink sites and assign each source to the nearest of these.

Section C

An Approximating Algorithm: The Minimum Elimination-Gain Algorithm (MEGA)

Instead of pursuing the solution of selection-allocation problems via the path of integer programming, we now turn our attention to an approximating algorithm which renders excellent local optima.

The idea behind this algorithm is very simply stated: Let each and every eligible sink be chosen, in violation of the stated goal of having only $m < S$ sinks. Then we may reduce this violation by eliminating one sink. The eliminant is the eligible sink whose elimination causes the value of the objective function

$$F = \sum_i \sum_j d_{ij} x_{ij}$$

to increase the least amount, thus the name minimum elimination-gain algorithm. This process is repeated on $S - 1$, $S - 2$, ... eligible sinks until $S - m$ sinks have been eliminated.

To make this process clearer we shall illustrate it with an example in which no ties occur between eliminants. Then we shall illustrate an iteration of another example in which tie-breaking is required.

Suppose we have seven sources and five eligible sink sites and the table of d_{ij} values is as shown in Table 1-4a. The initial allocation of sources to sinks is shown by the circle around the minimum d_{ij} in each row. We may read the table to see that

Source 1 goes to sink 1
Source 2 goes to sink 2
Source 3 goes to sink 3
Sources 4, 5, 6 go to sink 4
Source 7 goes to sink 5 .

Moreover, the value of the objective function is the sum of the circled elements or $F = 26$.

We now reason as follows: If any given column is eliminated with a loss of circled elements, then the circle must move to the next smallest element of the row in which the circle was lost. Thus, for example, if column 4 were to be eliminated three circled elements would be lost and we would have

Source 4 would go to sink 3 for a net gain of $3 = 13 - 10$
Source 5 would go to sink 5 for a net gain of $16 = 17 - 1$
Source 6 would go to sink 1 for a net gain of $12 = 19 - 7$

Total gain from eliminating column 4 = 31

SOURCES	ELEGIBLE SINKS 1	2	3	4	5
1	①$_{19}$	41	43	20	23
2	41	②$_{24}$	26	29	46
3	43	26	②$_{18}$	23	20
4	30	25	13	⑩$_{3}$	21
5	20	29	23	①$_{16}$	17
6	19	22	30	⑦$_{12}$	24
7	23	46	20	17	③$_{14}$
$G_j \rightarrow$	19	24	18	31	$\underline{\underline{14}}$

F = 26

Table 4a Selection-Allocation Problem, N=7, S=5. Initial Allocation

	1	2	3	4
1	①$_{19}$	41	43	20
2	41	②$_{24}$	26	29
3	43	26	②$_{21}$	23
4	30	25	13	⑩$_{3}$
5	20	29	23	①$_{19}$
6	19	22	30	⑦$_{12}$
7	23	46	20	⑰$_{3}$
$G_j \rightarrow$	$\underline{\underline{19}}$	24	21	37

F = 40 = 26 + 14

Table 4b Second Tableau N=7, S=5.

This is shown in Table 1-4a by subscripting each circled element by the gain in F which its loss would invoke. Obviously, the total gain in F resulting from the elimination of a column is the sum of these subscripts over the column. These sums, the elimination-gains, are shown at the bottom of the table labelled G_j . By what we have said above, we see that the first eliminant is eligible sink (column) 5. The result of this elimination is shown in Table 1-4b where we note that the elimination of a column also manifests its effect on the next table by taking away some second-bests.

Without prolonging the example we may look ahead to see that the final solution for $m = 2$ will be:

Source 2 to sink 2 .
All else to sink 4 .

Two kinds of ties occur. The first kind, that between elements of a row is of no consequence. When a source is indifferent to a group of sinks, choose one arbitrarily since the contribution to G_j will be zero no matter where the circle is until only one of this group is left.

The second kind of tie is between G_j's and is important. This situation is handled by the following addition to the rules (which will be restated more formally anon):

> In each column for which G_j is tied for minimality tag those elements which are second best of their respective rows. Compute a sum analogous to G_j based on the differences between third-bests and these second-bests. Use these sums in place of G_j to break the tie. If any second generation G_j's are still tied we repeat the process with third and fourth-bests, etc. until either the tie is broken or we exhaust the number of columns left. At this point we may try to break the tie on the basis of the largest sum of d_{ij}'s . If this fails, we choose arbitrarily.

An illustration of this tie breaking procedure is given in Table 1-5 which is a symmetric distance matrix. The minimal elements are again circled while the second-best elements are enclosed in squares. The subscripts on the circles are as before while the subscripts on the squares are the differences, third-best minus second-best. Note that the original and the tie-breaking sums are differentiated by a superscript.

SOURCE \ SINK	1	2	3	4	5	6	7
1	$\textcircled{0}_{14}$	24	$\boxed{14}_{10}$	26	38	30	54
2	24	$\textcircled{0}_{14}$	38	26	$\boxed{14}_{10}$	54	30
3	14	38	$\textcircled{0}_{12}$	$\boxed{12}_{2}$	24	16	40
4	26	26	$\boxed{12}_{0}$	$\textcircled{0}_{12}$	12	28	28
5	38	14	24	$\boxed{12}_{2}$	$\textcircled{0}_{12}$	40	16
6	30	54	$\boxed{16}_{8}$	28	40	$\textcircled{0}_{16}$	24
7	54	30	40	28	$\boxed{16}_{8}$	24	$\textcircled{0}_{16}$
G_j^1	14	14	12	12	12	16	16
G_j^2			18	$\underline{\underline{4}}$	18		

F = 0

Table 1-5 Illustration of Tie-Breaking

Before giving a precise, formal description of MEGA we pause to make the following observations:

(1) Note that a solution is given for every m', $m \leq m' \leq S$, hence the MEGA provides some insight into the cost of reducing the number of sinks, i.e. the G_j's may be regarded as analogous to the shadow prices of linear programming. The minimal G_j used in reducing the number of sinks from $k + 1$ to k is the added transportation cost invoked by this reduction. This must, of course, be weighed against the capital and operating costs involved in having a $k + 1$-st sink.

(2) A procedure which might produce even better solutions would be to allow an eliminant to replace a column (sink) still in any iteration numbered two more than the iteration on which it was originally removed. Thus in Table 5 we would consider trading column 4 for another column when the number of columns had been reduced to 5.

We now state the MEGA more formally by making the following definitions:

Let $J = \{j \mid j$ has not been eliminated$\}$,

$$\{\delta_i^k\} = \{d_{ij} \mid j \in J\}$$

but

$$\delta_i^1 \leq \delta_i^2 \leq \cdots \leq \delta_i^p$$

where p is the number of j's in J , and ties for precedence are broken on the basis of the smaller value of j .

Secondly, by the statement

$$d_{ij} \equiv \delta_i^k$$

we shall understand that δ_i^k is not merely equal to d_{ij} but is the identical element. This is an important distinction when more than one d_{ij} in a row has the same value.

Define $$g_{ij}^k = \delta_i^{k+1} - \delta_i^k \quad \text{if} \quad d_{ij} \equiv \delta_i^k$$

$$= 0 \qquad \text{otherwise} .$$

Finally, let

$$G_j^k = \sum_i g_{ij}^k \ . \qquad (3.6)$$

The Steps of MEGA

1. Set $p = S$, $J = \{1, 2, \ldots, S\}$.
2. Set $t = 1, T = J$.
3. For each $j \in T$ compute G_j^t .
4. If $\{G_j^t | j \in T\}$ has a unique minimum go to step 10; otherwise go to step 5.
5. $t + 1 \rightarrow t$; if $t = p$ go to step 8; otherwise go to step 6.
6. Redefine $\{T = \{j | G_j^{t-1}$ is minimal$\}$.
7. Go to step 3.
8. For each $j \in T$ compute $\sigma_j = \sum_i d_{ij}$ and define ℓ by $\sigma_\ell = \underset{j \in T}{\mathrm{Max}}\ \sigma_j$. If ℓ is not unique, define $\ell = \mathrm{Min}\{j | j \in T , \sigma_j$ is Maximal$\}$.
9. Go to step 11.
10. Define ℓ by $G_\ell^t = \underset{j \in T}{\mathrm{Min}}\ G_j^t$.
11. $J - \{\ell\} \rightarrow J$, $p - 1 \rightarrow p$.
12. If $p = m$ quit; otherwise go to step 2.

Section D

Applications of MEGA to Location-Allocation and Other Problems

We have noted above that Cooper [A 5], [A 6] solves location-allocation problems by an algorithm called the Destination Subset Algorithm. In this algorithm the locations of the sinks are restricted to the sources (Cooper's destinations) themselves. Cooper suggested that the $\binom{N}{m}$ possible solutions be generated.

It is apparent that picking the best m sinks from amongst N sources is a selection-allocation problem and is thus solvable by the MEGA. Table 1-5 is, as a matter of fact, derived from Cooper's second example (see Table 1-2) by using ℓ_1 rather than Euclidean-metric distances. Application of MEGA to Table 1-5 results in the ℓ_1-allocations shown in Table 1-2 one of which is identical with the E-allocation. It should be reiterated that the tableau handled by MEGA need not be square nor need it be symmetric.

CHAPTER 4

EXTENSIONS OF SELECTION-ALLOCATION MODELS

Section A

S-A Problems with Weighted Sources

In the location-allocation models and the selection-allocation models which we have thus far discussed sources have represented single truckloads of refuse. For many applications outside of the field of solid wastes it may be desirable to allow different amounts of the commodity of flow be present at the various sources. Within the field of solid wastes, advantages may be gained by taking many sources and representing them as a single source. Without committing oneself to a specific method for doing so, it can be seen that such aggregation allows larger areas to be analyzed; simplification is purchased at the price of an accuracy which may be inappropriate.

The mathematical statement of a weighted selection-allocation problem without capacity constraints is given by (3.5) with the following changes to the objective function, the P-system, and the coupling:

$$\text{Minimize} \quad \sum_{i=1}^{i=N} v_i \sum_{j=1}^{j=S} d_{ij} \, x_{ij}$$

$$\left.\begin{aligned} &\text{S.T.} \sum_{j=1}^{j=S} x_{ij} = 1 \quad \text{(no change)} \\ &\sum_{i=1}^{i=N} v_i \, x_{ij} - y_j = 0 \quad j = 1, 2, \ldots, S \\ &\sum_{j=1}^{j=S} y_j = V \end{aligned}\right\} P \qquad (4.1)$$

$$y_j \leq V \, \alpha_j \quad j = 1, 2, \ldots, S \quad \text{Coupling}$$

where v_i = weight at the i-th source

and $V = \sum_{i=1}^{i=N} v_i$.

There is no change to the A-R system.

The MEGA may be extended to cope with such a weighted problem. The only change required is in the key definition (3.6):

$$G_j^k = \sum_i v_i \; g_{ij}^k \tag{4.2}$$

This is made more concrete by the following example which is presented by Maranzana [A 24] as an example which causes his algorithm to end with a local optimum. The initial and the last two tableux are shown. Note that MEGA can be used to reflect analysis upon the weights themselves. For example, the next to the last tableau shows that eligible sink 7 becomes competitive with number 5 when the sum of the 6-th through the 9-th weights equals 20.

It should be noted that the example is especially constructed to have a self-evident optimum (2 and 5) and the local optimum found by Maranzana's algorithm (see page 1-16) is 5 and 7.

V_i	1	2	3	4	5	6	7	8	9
10	$\textcircled{0}_{10}$	10	20	20	30	140	130	140	140
100	10	$\textcircled{0}_{10}$	10	10	20	130	120	130	130
10	20	10	$\textcircled{0}_{10}$	20	30	140	130	140	140
10	20	10	20	$\textcircled{0}_{10}$	30	140	130	140	140
100	30	20	30	30	$\textcircled{0}_{20}$	110	100	110	110
1	140	130	140	140	110	$\textcircled{0}_{10}$	$\boxed{10}_{10}$	△20 $_{0}$	▽20 $_{90}$
1	130	120	130	130	100	$\boxed{10}_{0}$	$\textcircled{0}_{10}$	△10 $_{0}$	▽10 $_{90}$
1	140	130	140	140	110	△20 $_{0}$	$\boxed{10}_{10}$	$\textcircled{0}_{10}$	▽20 $_{90}$
1	140	130	140	140	110	△20 $_{0}$	$\boxed{10}_{10}$	▽20 $_{90}$	$\textcircled{0}_{10}$
G_j^1	100	1000	100	100	2000	10	10	10	10
G_j^2						0	30	0	0
G_j^3						0		0	0
G_j^4						0		90	270

$F = 0$

○ ≤ □ ≤ △ ≤ ▽

Table 1-6a MEGA with Weights on a Problem of Maranzana. Initial Tableau

V_l	2	5	7
10	$\textcircled{10}_{20}$	30	130
100	$\textcircled{0}_{20}$	20	120
10	$\textcircled{10}_{20}$	30	130
10	$\textcircled{10}_{20}$	30	130
100	20	$\textcircled{0}_{20}$	100
1	130	110	$\textcircled{10}_{100}$
1	120	100	$\textcircled{0}_{100}$
1	130	110	$\textcircled{10}_{100}$
1	130	110	$\textcircled{10}_{100}$
G_j^1	2600	2000	400

$F = 330$

Table 1-6b 7^{th} Tableau

2	5
$\textcircled{10}$	30
$\textcircled{0}$	20
$\textcircled{10}$	30
$\textcircled{10}$	30
20	$\textcircled{0}$
130	$\textcircled{110}$
120	$\textcircled{100}$
130	$\textcircled{110}$
130	$\textcircled{110}$

$F = 730 = 330 + 400$

Table 1-6c Final Tableau for M=2

Section B

S-A Problems with Site Acquisition and Processing Costs

In section C of Chapter 1-2 we gave an example of a fixed charge problem in the context of a problem in area exclusion. Having now passed to the ultimate in area exclusion, to wit, restriction to a finite set of points of known location, we turn to consideration of methods for making these models more realistic economically.

Thus far the only costs considered by the selection-allocation models have been the costs of transportation. This is given route-by-route in the coefficients d_{ij} (see page 1-79) with the aggregate transportation cost given by

$$F = \sum_{i=1}^{i=N} v_i \sum_{j=1}^{j=S} d_{ij} x_{ij} \qquad (4.3)$$

in the case of weighted sources. There are, however, two additional broad categories of cost which must enter into the economic design of a refuse system. These are the capital costs involved in acquiring and developing a refuse disposal site and secondly the costs of operating and maintaining the site. (It is often considered expedient if not appropriate to lump other system costs with the latter costs.)

To include these three categories of cost in the same analysis one must ascertain the proper scaling to be used amongst them so that one is not talking about one day's transportation cost versus one year's operation and maintenance. Assuming that the proper discounting factors have been computed and lumped into the coefficients about to be defined we let

K_j = Capital costs of acquiring and developing the j-th eligible sink site.

P_j = Cost per unit refuse processed at the j-th site when developed.

We may now include these cost considerations in the selection-allocation formulation of (4.1) and (3.5) by using the following objective function:

$$F = \sum_{i=1}^{i=N} v_i \sum_{j=1}^{j=S} d_{ij} \, x_{ij} + \sum_{j=1}^{j=S} (K_j \, \alpha_j + P_j \, y_j) \qquad (4.4)$$

Note that the capital cost K_j is not invoked unless the j-th site is accepted ($\alpha_j = 1$) and is invoked en toto or not at all. In such a context, the A-R system is not as fictional as first described. Note also that the processing cost, P_j is brought to bear on the full amount of refuse y_j arriving at the j-th site; allowance for partial operation may be included in the computation of P_j.

We end this section by showing how the MEGA may be extended to include the fixed charge or capital costs. The extension to account for operation and maintenance costs is not so easily made and is harder to justify. This will not be done here. The extension of MEGA to capital cost is accomplished by redefining

$$G_j^k = \sum_i v_i g_{ij}^k + \sum_{\ell \in J-j} K_\ell \qquad (4.5)$$

The rationale for this definition harks back to the intended meaning for the G_j^k's as originally defined, namely the amount by which F will increase if site j is eliminated, on the supposition that at each step all sites are in operation.

By way of illustration we may turn back to Table 6b and ask ourselves what K_2, K_5 and K_7 must be for site 5 to be preferable to site 7. Accordingly we have by (4.5):

$$G_2^1 = 2600 + K_5 + K_7$$

$$G_5^1 = 2000 + K_2 + K_7$$

and

$$G_7^1 = 400 + K_2 + K_5 .$$

Thus, we desire that

$$2600 + K_5 + K_7 > 2000 + K_2 + K_7$$

and

$$400 + K_2 + K_5 > 2000 + K_2 + K_7$$

or

$$600 > K_2 - K_5$$

and

$$K_5 - K_7 > 1600 .$$

Thus, for example if $(K_2, K_5, K_7) = (1300, 2500, 800)$ we have $G_2^1 = 5900$, $G_5^1 = 4100$, and $G_7^1 = 4200$ and site 5 is preferred over site 7.

Section C

Extended Models for Considering the Development and Use of a Selected Site

Once a selection-allocation problem has been solved by an exact algorithm or the MEGA, all we have done is find a set of refuse disposal sites and an allocation of service areas (represented by source points) thereto which minimizes the aggregate transportation cost. Using the extensions of the previous section it is possible to bring the costs of acquiring, developing and operating the site into the picture, but on closer examination, the manner in which this was done (see equation (4.4)) was a very rough cut at the economic analysis one desires in doing such planning. Clearly, the cost of acquiring a site and the cost of developing the site are separate items; lumping them together is made necessary by the limitations of the selection-allocation model.

What we seek then is a model or a class of models which, taking cognizance of the use to which a site will be put, is able to separate acquisition from development cost. It is clear that both development cost and operating costs depend upon the type and operating level of the facility developed at a site.

The problem of deciding what type of facility imposes an unwarranted and needless complication upon the analysis. The process of sifting available locations to determine the set of eligible sites to present to the analytic model rightly includes consideration of how any site may be used. That is, the analyst using his seasoned engineering judgment has probably decided already how **any given** site will be used. This is a qualitative judgment which may weigh intangibles **impossible** or difficult to quantify and model.

On the other hand, given the type of facility to be erected on a site, there are latitudes of choice open in determining how large, how fast, how expandable, etc. to make a facility at any given site. This is an area of quantitative decision making where computer-oriented analyses can aid human judgment.

If, for example, a site is suitable for an incinerator, the site development choices may represent several combinations of capacity and processing rate of interest to the planner. The planner has the responsibility for developing a reasonable number of realistic alternatives which covers the range of practical interest for the site, locality, etc.

Here, again, we are opting for simplicity by laying a equitable burden upon the planner. The alternative is to consider continuous variation of design parameters. This is possible in the design of a single facility, but may be exceedingly complex in a system of interrelated facilities.

A model for the analysis of the acquisition, development, and operation of multiple sites is illustrated in Figures 1-11a and 1-11b. Here we are selecting two sites ($m = 2$) out of four eligible sites ($S = 4$). Sites 1 and 2 are each developable in three ways while sites 3 and 4 are developable in four ways each ($W_1 = 3$, $W_2 = 3$, $W_3 = 4$, $W_4 = 4$) . **Figure 1-11a presents** the analog of the A-R system for the selection-allocation model. The left half of the network is familiar with the A node, nodes 1, 2, 3, and 4 and the variables $\alpha_j (j = 1, 2, 3, 4)$ retaining their former meaning, but the R part of the network has been replaced by a system of doubly-indexed flows δ_{jk} representing choice of the k-th development of the j-th eligible site. Figure 1-11b represents the physical or P system. Here again, the flow from **an** eligible site is split into different flows on an all or nothing basis amongst the different development possibilities.

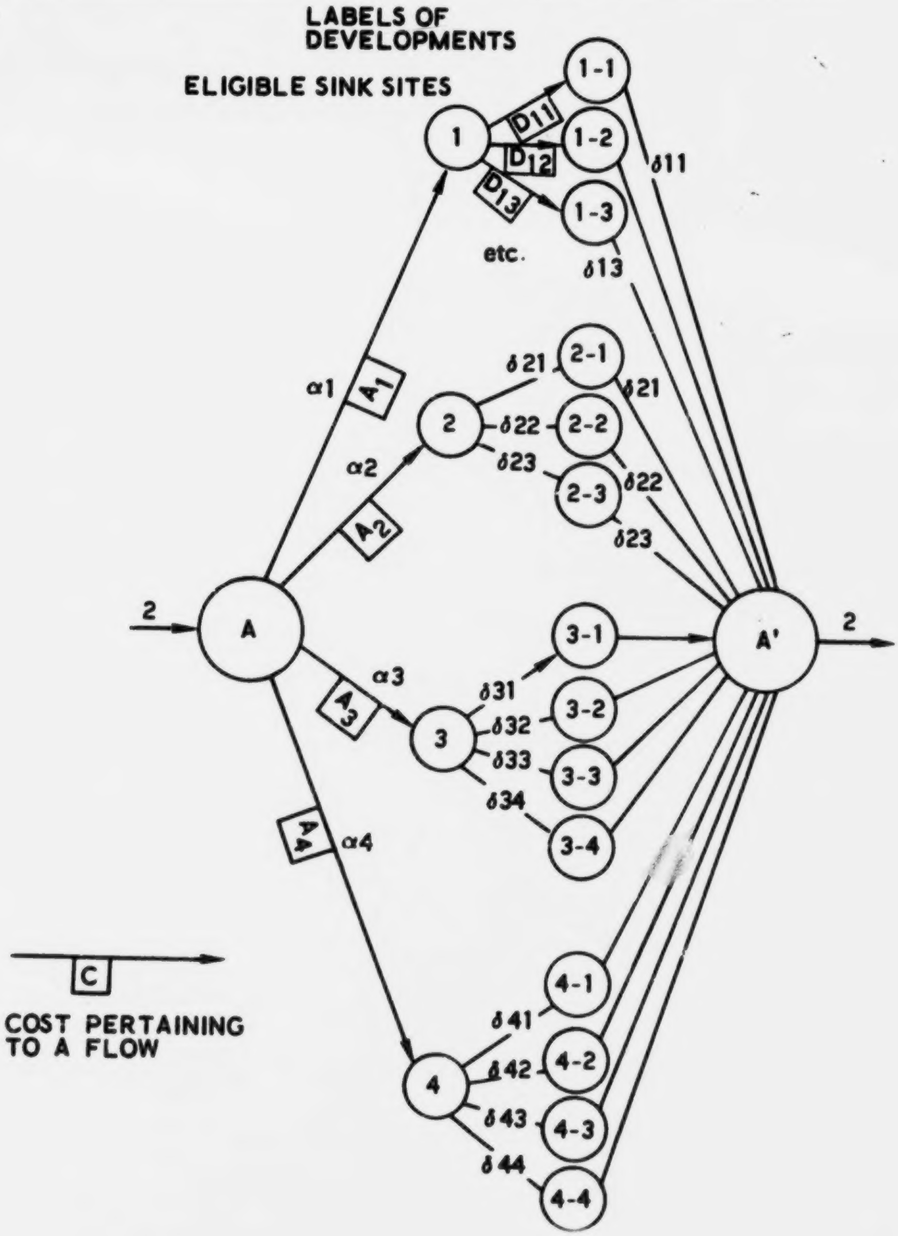

LABELS OF DEVELOPMENTS
ELIGIBLE SINK SITES
1-1
1-2
1-3
1
D11
D12
D13
δ11
etc.
δ13
α1
A1
2-1
δ21
δ21
2
δ22
2-2
δ23
2-3
δ22
α2
A2
δ23
2
A
A'
2
3-1
α3
A3
δ31
3-2
3
δ32
δ33
3-3
δ34
3-4
A4
α4
C
COST PERTAINING TO A FLOW
4-1
δ41
4-2
4
δ42
δ43
4-3
δ44
4-4

Figure 1-11a Acceptance System Network for Extended Selection Model

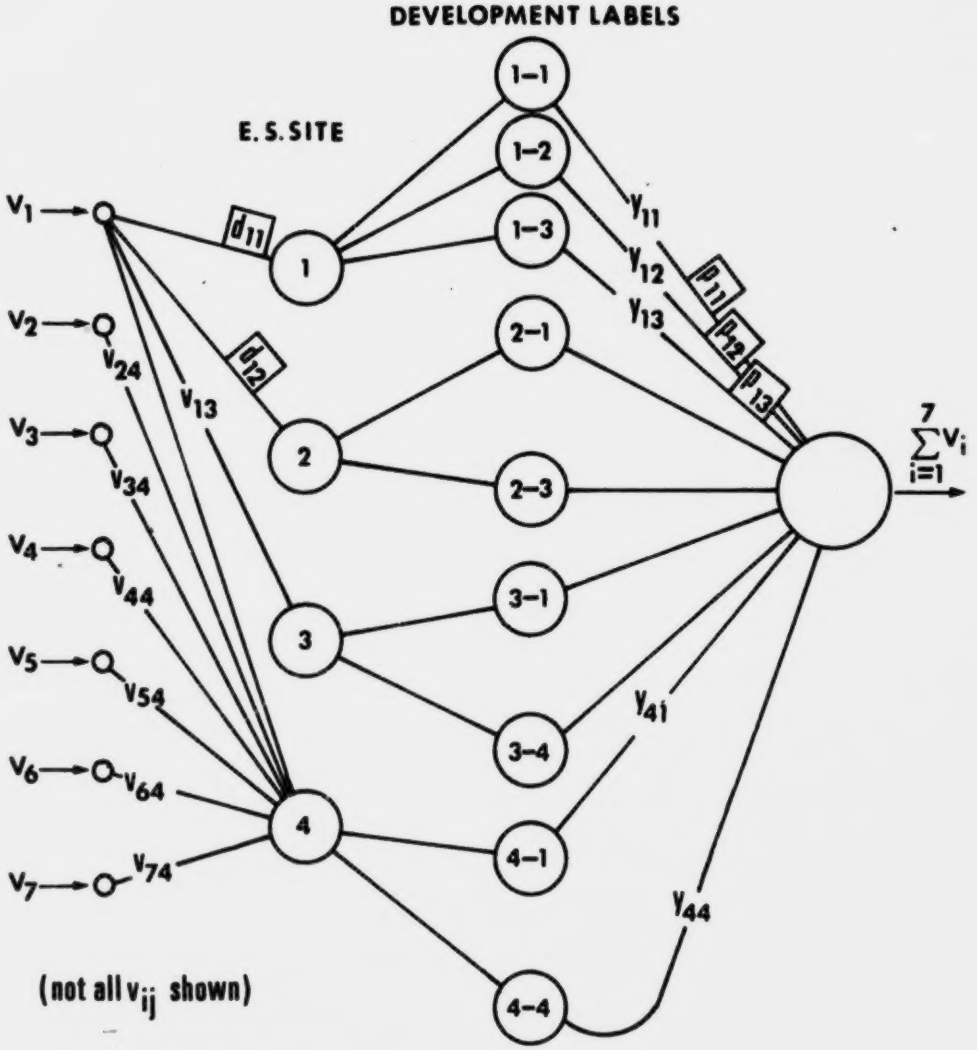

DEVELOPMENT LABELS
E. S. SITE
v_1
v_2
v_3
v_4
v_5
v_6
v_7
d_{11}
d_{12}
v_{13}
v_{24}
v_{34}
v_{44}
v_{54}
v_{64}
v_{74}
1
2
3
4
1-1
1-2
1-3
2-1
2-3
3-1
3-4
4-1
4-4
y_{11}
y_{12}
y_{13}
p_{11}
p_{12}
p_{13}
y_{4i}
y_{44}
$\sum_{i=1}^{7} v_i$
(not all v_{ij} shown)

Figure 1-11b Physical Network of Extended Selection Model

To make this more precise, we define the following variables and constants:

V_i = Amount of refuse at the i-th source.

v_{ij} = Flow sent from the i-th source to the j-th site. We are reverting to a continuous variable formulation since capacity constraints may be relevant in the choice of a development.

d_{ij} = Cost of transporting one unit (e.g. ton) of refuse from the i-th source to the j-th sink.

y_{jk} = Amount of refuse processed by the k-th development at the j-th site.

p_{jk} = Processing cost per unit refuse using the k-th development at the j-th site. (Operation and maintenance, etc.).

δ_{jk} = 1 if the k-th development at the j-th site is undertaken

= 0 otherwise.

D_{jk} = Cost of undertaking the k-th development at the j-th site.

α_j = 1 if the j-th site is chosen

= 0 otherwise.

A_j = Cost of acquiring the j-th site.

We are assuming here that the various costs are suitably scaled (discounted) with respect to one another so that the component parts of the objective function can be added realistically. The overall model is then

$$\text{Minimize} \quad \sum_i \sum_j d_{ij} v_{ij} + \sum_j \sum_k (p_{jk} y_{jk} + D_{jk} \delta_{jk}) + \sum_j A_j \alpha_j$$

subject to

$$\left.\begin{array}{l:l}
\displaystyle\sum_{j=1}^{S} v_{ij} = V_i & \displaystyle\sum_j \alpha_j = m \\
\displaystyle\sum_k y_{jk} = \sum_i v_{ij} & \displaystyle\sum_k \delta_{jk} = \alpha_j \\
\displaystyle\sum_j \sum_k y_{jk} = \sum_i V_i & \displaystyle\sum_j \sum_k \delta_{jk} = m \\
\hdashline
\end{array}\right. \qquad (4.6)$$

$$0 \leq y_{jk} \leq \delta_{jk} \sum_i V_i \; .$$

1) All variables non-negative and α_j , $\delta_{jk} = 0$ or 1 only.

2) m = Number of sites to be selected.

3) All summations over i are from 1 to N , the number of sources. All summations over j are from 1 to S the number of eligible sink sites. All summations over k are from 1 to W_j the number of ways to develop the j-th site.

In addition to the constraints shown in (4.6) it is quite likely that capacity constraints would be relevant. A simple capacity constraint of the form

$$\sum_i v_{ij} \leq C_j \qquad (4.7)$$

is easily accomodated, but what is more likely is a compound capacity constraint which says

If $0 \leq \sum_i v_{ij} \leq C_{j1}$ use development 1 .

If $C_{j1} \leq \sum_i v_{ij} \leq C_{j2}$ use development 2 .

$\vdots$

etc.

Such a system of capacity constraints can be imposed by altering the coupling constraints. For example, if a set of capacity constraints of this nature were to be imposed on the first site of the example of Figure 11, the coupling constraints might read

$$0 \leq y_{11} \leq C_{11}\delta_{11}$$

$$C_{11}\delta_{12} \leq y_{12} \leq C_{12}\delta_{12}$$

$$C_{12}\delta_{13} \leq y_{13} \leq C_{13}\delta_{13} .$$

CHAPTER 5

SUMMARY AND CONCLUSIONS

The motivation for and the course taken by the investigations of Part I have revolved about the answers to two questions:

1) Where is the best place to locate one or more refuse disposal sites given the locations of the areas to be served?
2) Which areas should be served by which disposal site?

We began our answering of these questions by making several assumptions. We assumed, most fundamentally, that one plan for locating the disposal sites and allocating the service areas would be distinguished from another such plan on the criterion of aggregate transportation distance of service areas to disposal sites. We assumed, further, that there would be no capacity constraints at any disposal site and that distances would be measured by the ℓ_1-metric.

Under these assumptions we were able to formulate the location-allocation problem as a linear program involving integer variables. Then, taking advantage of the median nature of pure location problem solutions when using the ℓ_1-metric, we constructed a theory of median sets which was used to support an algorithm for the approximate solution of location-allocation problems. This algorithm operated by alternately locating and allocating.

Up to this point we had been assuming that the disposal sites could be located anywhere in the plane. Thus, we turned to the question:

3) What happens to the choices of disposal site location and the allocation of service areas when certain areas are ineligible for the location of disposal sites?

For simple cases, we demonstrated both graphical and analytic solutions. For a problem involving the location of a single disposal site, the best location is that location in permitted territory ℓ_1-closest to the unrestricted optimum location.

Instead of pursuing this question for the problem of locating several disposal sites, we passed question 3) to the limit: the process of declaring certain areas ineligible culminates in the situation wherein only a small number of land parcels remain eligible. The question then becomes:

4) Given a definite number of eligible sites and their physical locations, what is the best way to choose a proper subset (i.e. some but not all) of them to be disposal sites and to allocate service areas to them?

Whereas the former models were called location-allocation problems, we termed the models resulting from question 4) "selection-allocation" problems. The common features of the two classes of models are the problem of allocation and the goal of minimizing aggregate transportation distance. In comparing the two classes of models, one may conclude that the location-allocation problem is likely to be more of an interesting mathematical abstraction than a useful technique. The real-world situation facing the planner bears a stronger resemblence to the situation underlying the selection-allocation problem.

Again, we were able to characterize the selection-allocation problem as a linear programming problem with integer variables. The transition to a linear program was aided by a network-flow analog and did not depend on any particular metric for giving the distances. Thus, assumption of ℓ_1-metric distances (which was necessary for the linearization of the location-allocation problem) was dropped. As before, an approximating algorithm was developed. This algorithm, termed the *minimum elimination gain algorithm* (MEGA), assumes that all eligible sites are used to start. These are eliminated one-by-one until the requisite number remain. The basis for elimination is: eliminate that site whose elimination causes the minimum gain (increase) in the aggregate distance function.

The final section of Part I attempted to exploit the flexibility of the selection-allocation model to introduce further elements of realism. The first such extension allowed the service areas to be weighted according to relative activity. It was found that the MEGA was useful in this extension. Next, and more important, we investigated the extension of selection-allocation models to cover site acquisition, development and operating costs. Several models were proposed as integer-linear programs. Only the isolated problem of site acquisition cost could be handled by the MEGA.

To extend the remarks made above concerning comparison between the location-allocation models and selection-allocation models, the writer ventures to speculate that the extended models which encompass acquisition, development, and operating costs as well as aggregate transportation cost will prove to be valuable planning tools when fully and intelligently exploited. It is appropriate to repeat here that intelligent exploitation of mathematical models puts a great burden on the planner to make those choices and eliminate those alternatives which are within the realm of mature engineering judgment. Intelligent exploitation of mathematical models regards the latter as an aid to, not a substitute for, such judgment.

PART II

ON THE SIMULATION OF REFUSE COLLECTION AND HAUL OPERATIONS

CHAPTER 1

INTRODUCTION

Section A

Solid Wastes Literature

In this section it is our intent to review the scope and accomplishments of work done in the field of solid wastes. More particularly we shall concentrate on investigative methods that are quantitative, mathematical and/or analytic for the most part and on work of the last thirty years. It would not be proper, however, to dismiss preceding work without at least a cursory sketch.

The solution to the problems of refuse collection and disposal was simple in the civilizations of antiquity and remained essentially the same into modern times where populations were rural, agrarian and sparse. The solution consisted of selecting a place removed from the village (presumably downwind) and dumping the refuse there on the open ground. Such dumping grounds have proved the treasure troves of modern archeologists. The responsibility for refuse removal was on the individual family in small settlements and we may assume that some sort of organized activity manned by slaves was used in the cities of antiquity.

A modern literature on refuse begins sometime in the nineteenth century with the growth of organized municipal services in all areas affecting the public health. Such literature was and still is the literature produced by and for day-to-day practitioners in the field of public work or by engineers reporting on specific solutions to specific problems. Modern examples of such literature are found in publications such as American Gas Monthly (articles concerning use of gas as an auxillary fuel in refuse incineration), Engineering News (design of incinerators and other facilities), Power Engineering, Gas Age, Combustion, Plant Engineering, Public Works (by and for public works officials and functionaries), Refuse Removal Journal (the same, only more specifically toward those in solid wastes), Sewage and Industrial Wastes (concerned with solid wastes produced by industries), and finally Water Works Engineering (where solid wastes interface with water supply and treatment). No specific works from these journals are cited herein.

Probably the cornerstones of recent work in the field of solid wastes are two volumes published by the American Public Works Association, Refuse Collection

Practice [5] first published in 1941 and revised in 1958, and Municipal Refuse Disposal [4], 1961. The first of these deals exhaustively and systematically with all aspects of refuse collection starting with a statement of the problem, an examination of the types and amounts of substances aggregatively called refuse, the role of the householder (or other producers) in preparation of refuse for collection, analysis of the costs of refuse collection, survey of methods of collection and types of equipment, and going on to discuss financing and multifarious aspects of operating and administering such systems. We shall have more to say anon on methods of planning refuse collection systems described by this book.

Municipal Refuse Disposal concentrates more on the description and analysis of different methods of ultimate disposal, their advantages and disadvantages and their interaction with various aspects of the collection system. Both of these books contain excellent selected bibliographies.

The thing to be noted about both of these books is that all of the identifiable subcomponents of the total system are discussed and analyzed separately. Frequently, this analysis involves equations or formulas, but when it comes to describing the interaction of these parts, the description is more likely to be the qualitative description of tendencies than the quantitative prediction of formulas.

The first large-scale effort to combine the work study method of industrial engineering with an overall quantitative rational analysis of an entire refuse collection and haul system was undertaken by the Sanitary Engineering Research Project of the University of California (Berkeley). [1][1/] . Among the factors entering this analysis were pickup efficiency measured inversely by man minutes per ton, crew size, haul distance to the disposal site (from the center of the city), percent of pickup made from curb versus percent from rear of house (assuming blocks bisected by alleys), etc. A more extensive listing, definition and discussion of these factors and an examination and critique of these methods of analysis will be undertaken in the next chapter,[2/] but it should be noted here that this work established the definitions and concepts upon which this and much present-day work is based. Indeed, the Sanitary Engineering Research Laboratory of the University of California, (Richmond) is still a primary

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[1/] This is also reference Pr. 1 of the Preface.

[2/] A synopsis of the data and analytical techniques developed in this report are included as Appendix D to Refuse Collection Practice [5].

source of research and trained personnel in the area of solid wastes. (See below).

It has often been remarked that a great deal of what's new in America starts on the West Coast. This **certainly** seems to be true when applied to the growing awareness of such things as air pollution and ghetto pollution (Watts). In the area of solid wastes, the seed planted by the Richmond group took root in the Los Angeles area.

In the late 50's, early 60's, Los Angeles was faced with massive planning and design problems imposed by vast growth over a huge geographical area and compounded by the need to end backyard incineration of refuse in order to ameliorate the growing problem of air pollution. Various techniques such as statistical analysis, queuing theory and more recently, following lead of the research reported hereinunder, Monte Carlo simulation have been applied to problems such as the location and design of transfer stations and fleet maintenance reserve [7]. The Los Angeles group is also pioneering in the design of automated data gathering mechanisms to be installed on refuse collection vehicles3/ ; the underlying need of all techniques is good data.

3/ Personal communication.

Another application of system analytic technique that can be described in more detail concerns the City of Vancouver, B.C. [8]. The acquisition of new equipment of larger capacity necessitated the coalescence of the more numerous, smaller collection routes ("beats" in the language of the report) into fewer, larger ones. The study began with the gathering of detailed data on each block of the city. This data was used to get a regression formula relating time of collection to such factors as volume of refuse, numbers of containers, number of container groups, travel distance around the block, etc. Then the transportation algorithm of linear programming was used to balance the workload between collection routes. The analog of the suppliers was found in truckless or overloaded routes, the role of the receivers was taken by underworked trucks, while the commodity being "transported" was city blocks. The objective function being minimized reflected the number of changes from the existing routing, i.e. the number of blocks previously on the same route now on different routes. Thus,the overall objective was to minimize the disruption needed to achieve an equitable balance of workload within the constraints of volume, length of workday and physical compactness.

Among other work preceding 1965 we mention also the pioneer work of Charnes, Quon, and Wersan ([12] - [18]) part of which is extended in this dissertation. And as evidence of growing awareness and desire to do something we have already noted the work of the Northeastern Illinois Metropolitan Area Planning Commission (Reference Pr. 13 of the Preface) and the A.P.W.A.'s Solid Waste Research Needs (Reference Pr. 11 of the Preface) which is in effect an agenda of the research now in progress in many places.

Research in solid wastes increased immensely in the years 1961 - 1965 because of an increasing willingness of the United States Public Health Service and the National Institutes of Health to sponsor and support such research[4/]. Nonetheless, the year 1965 represents a cleavage between two research methodologies. Prior to 1965 one saw first the gradual emergence of the problem, pursued initially without urgency or pressure. Then a few researchers became concerned and began pioneer investigative work on limited funds and with scant communication so that the work that emerged was marked by large

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[4/] The work underlying this dissertation was so supported and this is acknowledged elsewhere.

gaps and some duplication. By the early 1960's the problem of solid wastes had become so universally recognized that funds became available, but the need and the clamor were for immediate solutions, not research. In order to provide for an orderly direction of future research, Congress passed the Solid Waste Disposal Act of 1965. It was hoped that this act would buttress a more sophisticated research methodology: one in which the environmental objectives of wastes management are first delineated in public policy. Then the problems which must be solved to meet these objectives are tackled in a coordinated manner and there exists a yardstick against which to measure proposed research.

A summary of research which has been performed largely under the aegis of this act was provided by a conference held in Milwaukee, Wisconsin in July 1967 [6][5/].
A brief description of this work follows:

Working with the City of Berkeley, California, C. G. Golueke is developing an overall "solid wastes system evaluation model" [6-A1]. Although not spelled out in detail in the conference proceedings, a part of this work entails the development of a refuse generation model.

[5/] The 59 papers are numbered A1-A11, B1-B12, etc. For the sake of brevity we shall list all the relevant papers under the same bibliographical heading and refer to them as [6-A1], [6-D4], etc.

This means being able to predict the amount of refuse produced (or to generate a random variable representing same) given all the relevant factors which affect refuse production. Since a refuse generation model is part of the work to be described hereinunder we shall defer further discussion of the subject to **Chapter 3**, but it is àpropos to note that the problem is of wide concern as evidenced by papers [6-A7, D1, D2, D4, D5, F3, F4, F5, F6].

Other investigative groups seeking to create overall systems evaluation models are the Aerojet General Corp. group [6-A3] which is putting its emphasis on development of measures of effectiveness, a group at the University of Southern California [6-A11], and finally the City of Los Angeles which is developing a management information system aimed at the supervision of operations and which as noted previously has used and is using such techniques as linear programming, queuing theory and Monte Carlo simulation [6-D1]. In the City of Baltimore, Maryland a group at The John Hopkins University is developing a Monte Carlo model for the design of a refuse collection system employing transfer stations [6-D2] while at Harvard the use of shipborne incinerators is being investigated by the use of a simulation model [6-D5].

In summary, work in progress falls into the following categories:

1. Investigation and improvement of the technology of current disposal techniques.
2. Investigation of proposed new disposal techniques (shipborne incineration, refuse pipeline, etc.).
3. Creation of an improved data base (refuse generation models, etc.).
4. Creation of a management and planning science for **solid** wastes through the use of computer and system analytic techniques (which is not confined to this category).

This appears to be an appropriate point to inject some personal comment. This author has come to feel that a part of the research money that the Federal government is putting into the problems of solid wastes would be more wisely spent on investigation of the economic and social incentives needed to

(a) Decrease refuse production.

(b) Salvage useful materials from refuse.

Point (a) is based on the real observation (cited below in its proper place) that people tend to produce more refuse when more frequent and more inclusive (of refuse types) service is provided. Faced with a more restrictive service, old clothes get saved for Goodwill, newspapers for the Boy Scouts, etc. Thus, the economic problem of trying to salvage such material from refuse is solved by providing an incentive that keeps them out of the refuse container in the first place. Another example that readily springs to mind is the proliferation of no-return beverage containers. If man is to learn to live cooperatively with his environment rather than disasterously, he may have to sacrifice many such wasteful conveniences. In the end, it boils down to whether government is to forever pander to unbridled demands for bigger, better and more by spending more and more money less and less effectively. No one denies that the Federal government or the several states now have the power to take such strong measures as banning beer cans, but Americans have a nortorious habit of not doing anything until a gun is against their temple.

Point (b) is based on recent reports of success in coking automobile tires and domestic refuse to produce large quantities of fuel gas and tar-like chemicals which can provide the raw materials for plastics, drugs, etc. Such methods, if they can be exploited without air pollution hazard, combined with incentives and regulations which reduce the amounts of refuse can well be the key to the future in this area.

Section B

The Nature and Limitations of Simulation

As we have noted above, Monte Carlo simulation has become a widely used technique since it was first reported in the works of Charnes, Quon, Eisen, Tanka, and Wersan ([12]-[18]) hereinunder extended. We have also noted that simulation is one of a variety of techniques intended to provide insight and predictive power in dynamic, random, changing environments. By way of contrast and introduction, then, we propose first to expand upon the list of study methods proposed in the Preface, turning thence to an extensive examination of the variety of techniques which collectively have come to be called "Monte Carlo methods".

First among investigative methods that might have an intuitive appeal to a planner wishing to ask a variety of "what if" questions would be to perform experiments with the entities involved. To a limited extent this has been, is being and will continue to be done. Thus, for example, in seeking ways to control the fly breeding problem in refuse awaiting collection, Ecke, *et al*. performed actual experiments with two different frequencies of collection against two different containerization methods: (once weekly, twice weekly) × (cans, suspended paper bags) [6-F6]. Moreover, the traditional method of balancing workload over collection routes has been by trial and error experimentation with crews and equipment ([5], p. 203), but the scope of questions that can be answered by experimentation is limited. And these limits exclude arbitrary purchase of equipment, large capital expenditures, hiring or firing of personnel or massive tinkering with the public's patience in varying frequency and/or site of collection.

The next approach on some scale of intuitive appeal is to use someone else's experience to represent an experiment on our part; i.e.,reasoning by anology. One finds a city of similar population, occupation, etc. and notes that what was good for it may be good for us. A little more tenuous than this is the use of historic analogy; failing to find a contemporary analog, one seeks a city which was some years ago in a situation similar to our present situation. Probably, the best place for "data" of this kind is as incidental backup for plans underwritten by more solid forms of analysis, or perhaps as a goad to civic pride when bond referenda are at stake. When used as the primary means of analysis, too much time is later spent figuring out why the analogy didn't hold after all.

The third general category of methods used for studying dynamic systems is a large one subsuming a wide variety of mathematical techniques. Proceeding from simple to complex we have first mathematical methods of classical engineering analysis such as statistical analysis (n-year flood, n-year storm) for the study of both long and short term variation and the marriage of the classical engineering methods of analysis via differential equations with probability theory which has come to be known as queuing theory (study of short term random demand on waiting line facilities). Here also we have the use of current demand, population prediction models, economic factors and over-design factors to yield facilities designed for a certain "design life". Certain rational methods of this type have been developed for the design of refuse collection systems and these will be discussed more fully in the next chapter.

The models of Part I, more especially of Chapter I-4, are an attempt to provide a system model which ties together a great many factors simultaneously, but as we have noted these models are simplifications of a dynamic world. Some insight into the problems engendered by long-term change can be gotten by solving these models repeatedly with the cost and refuse production data inputs that we <ins>expect</ins> will hold at various times in the future (sensitivity analysis). The problem here, of course, is that much is yet to be learned about the creation of adequate models of both population growth and <ins>present</ins> refuse production hence models of future refuse production are doubly hazardous. (See [11] for a discussion of demographic models.)

An alternative to running a time-fixed model many times is to create a model which allows planning over a time horizon, i.e., we have a large model whose subcomponents are the time-fixed models. These are coupled by constraints representing intertemporal conservation of matter (etc.) and imposition of common sense. Thus a variable which previously specified whether or not to acquire a given site is now represented by the sum of time subscripted variables which specify when to acquire that site (if at all). The coupling constraints stipulate such rules as "if acquired, it is acquired only once," "if acquired and/or operating in time period t it must operate in subsequent periods (unless filled up as a sanitary landfill site)", etc. The shortcomings of such an approach are first its overweening complexity and probable exhaustion of computer capacity, and second, it also requires the use of present-day guesses about tomorrow's data. In the case of site selection this entails the assumption that sites available now will continue to be or requires us to surmise which sites will become availabe that aren't now, etc. A part of the problem of future data can be obviated by running models with future data at low and high levels to study the range of outcomes.

Although used by some to study long term growth and change, the greatest use of simulation has been and is to study short term random and cyclical variation. In this day simulation has come to be conducted chiefly by the use of computer programs, but we may note that simulation is really an old technique and probably one of the oldest learning exercises known to man. ("Let's play house.") In the field of technology, simulation has such antecedents as basin testing of physical models of ships, wind-tunnel testing of airframes and airfoils, static loading models of structures and the use of analog computers. (See, for example, references A9, A10 and A27 of Part I.) But as applied to modern research in the social and physical sciences, simulation has come to mean:

> "the construction and manipulation of an operating model, that model being a physical or symbolic

> representation of some or all aspects of a complex system or process." [10][6]

Such operating models are "representations of behaving systems that attempt to reproduce the processes in action" [10][6]. To illustrate these ideas and to motivate further abstract discussion let us examine several situations which might be studied by simulation.

Example 1: Suppose we lay out a sheet of paper with parallel lines of spacing W and begin dropping a needle of length L onto it. Each time we do so we tally up the trial number and whether or not the needle crossed one of the lines on the paper. After many trials[7], the ratio of the number of crossings to total trials may be taken to approximate the probability that the needle will cross the line.[8] The flowchart for a program to simulate this experiment is given in Figure 2-1. It should be noted that in this example the computer is being used as a device that quickly generates random numbers[9]; there

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[6] Italics added.

[7] Just how many is a matter of statistical analysis and familiarity with a particular model. This is usually a matter of art rather than science.

[8] This is the classical problem known as Buffon's Needle. For a judicious choice of the ratio L/W , the experiment can be used to compute π .

[9] About which much has been written.

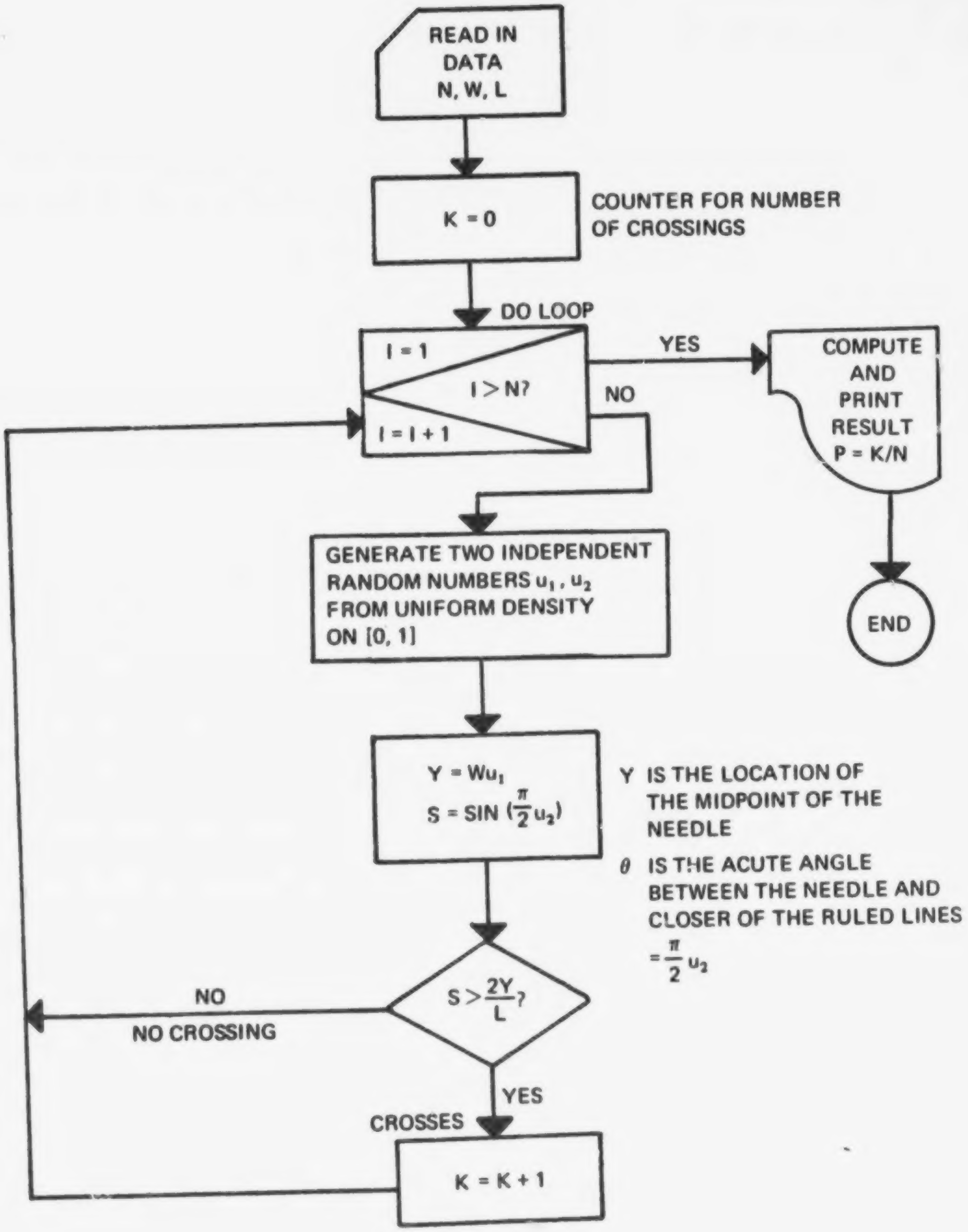
READ IN DATA N, W, L
K = 0
COUNTER FOR NUMBER OF CROSSINGS
DO LOOP
I = 1
I > N?
I = I + 1
YES
NO
COMPUTE AND PRINT RESULT P = K/N
END
GENERATE TWO INDEPENDENT RANDOM NUMBERS u_1, u_2 FROM UNIFORM DENSITY ON [0, 1]
$Y = Wu_1$
$S = \sin(\frac{\pi}{2}u_2)$
Y IS THE LOCATION OF THE MIDPOINT OF THE NEEDLE
θ IS THE ACUTE ANGLE BETWEEN THE NEEDLE AND CLOSER OF THE RULED LINES $= \frac{\pi}{2}u_2$
$S > \frac{2Y}{L}$?
NO
NO CROSSING
YES
CROSSES
K = K + 1

Figure 2-1 Flowchart for Simulating Buffon's Needle Experiment

is nothing in the problem that precludes manual simulation or theoretical analysis.

Example 2: In contrast to this simple problem, we might consider the solitaire-like card game known as "clock". In this game 13 piles of four cards each are dealt out, face down, 12 of them corresponding to the hours on the clock and the other at the center. A card is turned up on the center pile and is placed on the pile corresponding to its value (A = 1, J = 11, Q = 12, K = 13; K goes to center pile) and the top card of that pile is then turned up, etc. The game stops when all four kings are on the center pile and the "penalty" is the number of unturned cards remaining.

Suppose now that we limit our inquiry to finding the value of P[Kd = 0] where Kd is the number of remaining cards. A small amount of reflection shows: 1) P[Kd = 0] is so small that an extraordinarily large number of games would have to be played manually to determine it, 2) the determination of P[Kd = 0] is nearly impossible by theoretical means. But we do note that the event Kd = 0 is impossible if there isn't at least one king on the bottom of a pile as originally dealt. Thus, if

Kb = Event that at least one king is on the bottom of a pile .

$$P[Kd = 0] = P[Kd = 0 \cap Kb] = P[Kd = 0|Kb]\ P[Kb]$$

Thus, the analysis via a simulation program can be sharpened (meaning fewer trials) if we compute instead $P[Kd = 0|Kb]$ and multiply it by $P[Kb]$ which can be computed theoretically.[10] Thus, in the flowchart shown in Figure 2-2, the initial box marked "DEAL" is a process of randomly placing a king on the bottom of a pile and then randomly placing the other 51 cards. This deal is referred to in the flowchart as the array (N_{ij}) with four rows and thirteen columns with the values $1 \leq N_{ij} \leq 13$. We note next that the game may be likened to a treasure hunt in which there are four clues at each of thirteen caches. Each clue tells which cache contains the next clue. The rules stipulate that a clue may be used only once, one may not peek at a clue out of sequence and the game ends when we seek a fifth clue from the initial cache. Thus, in simulating "clock" it is immaterial which pile is considered initial (we have used the ace pile), and second, we do not need to simulate the physical movement of cards; a card stays where it was originally dealt but is marked as used.

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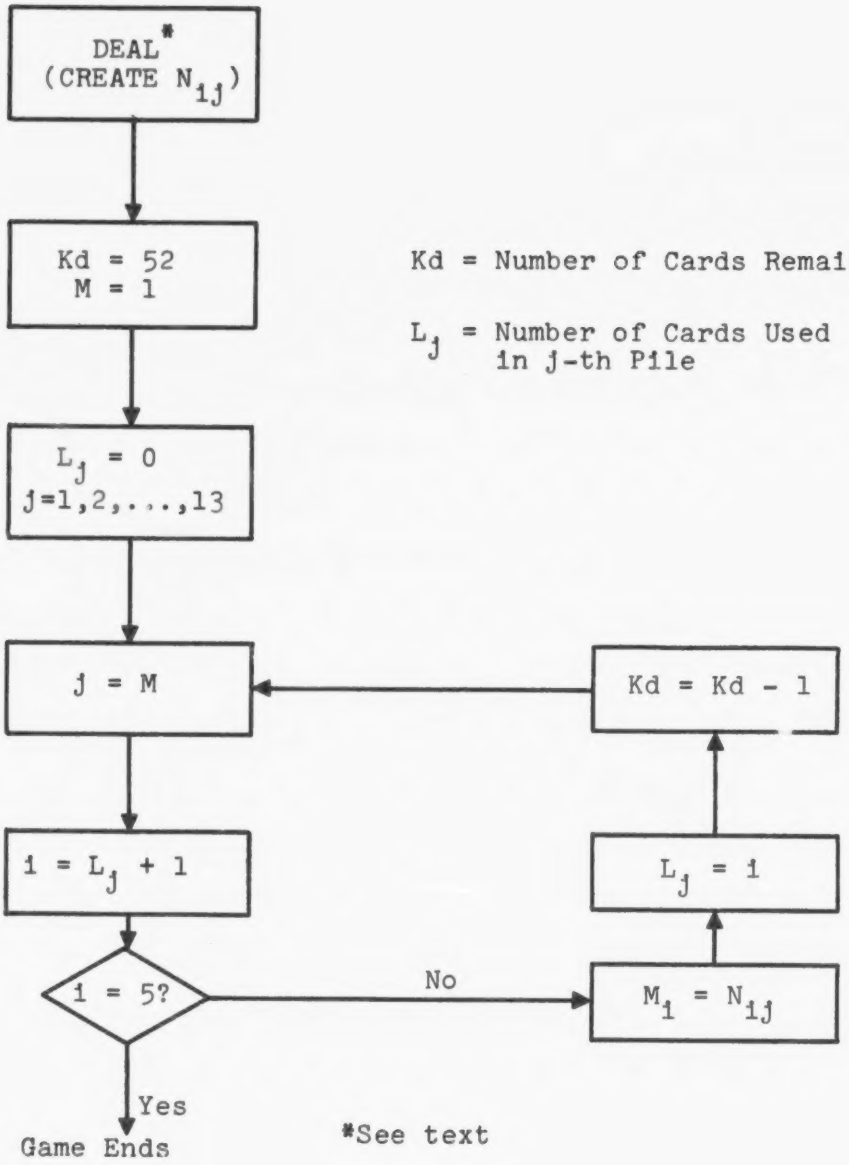
[10] $P[Kb] = .6961$.


DEAL*
(CREATE N_{ij})
$Kd = 52$
$M = 1$
$L_j = 0$
$j=1,2,\ldots,13$
$j = M$
$Kd = Kd - 1$
$i = L_j + 1$
$L_j = i$
$i = 5?$
No
$M_i = N_{ij}$
Yes
Game Ends

Kd = Number of Cards Remaining

L_j = Number of Cards Used in j-th Pile

*See text

Figure 2-2 Flowchart of One Hand of the Game of "Clock"

This example indicates that a simulation need not be a slavish imitation of every detail of the situation being modeled. On the other hand, the particular situation being modeled here is rather static; once the cards are dealt there can be but one outcome. Thus the function of the program is to discover this outcome swiftly.

Example 3: Suppose at time $t = 0$ that a drunk is supporting a lamp post located at $(0, 0)$ in a coordinate system. At times $t = 1, 2, 3, \ldots$ he makes a random move of one unit (step) north, east, south or west with equal probabilities of $1/4$ from the position occupied at time $t - 1$. We now have a situation which is thoroughly random; there is no deterministic deal at the beginning of the game. There is rather, a "game tree" with each node allowing four branches (Figure 2-3). Of course, many sequences such as $+ x$, $- x$ and $+ y$, $- y$ lead one back to the starting position. Among the things we might want to know is: given a bar at $(m, n) \neq (0, 0)$ how long will it take on the average for the drunk to get help for the lamppost? This is a simple example of a class of problems called "random walk problems" the solutions to which are used in such diverse fields as solving partial differential equations. The

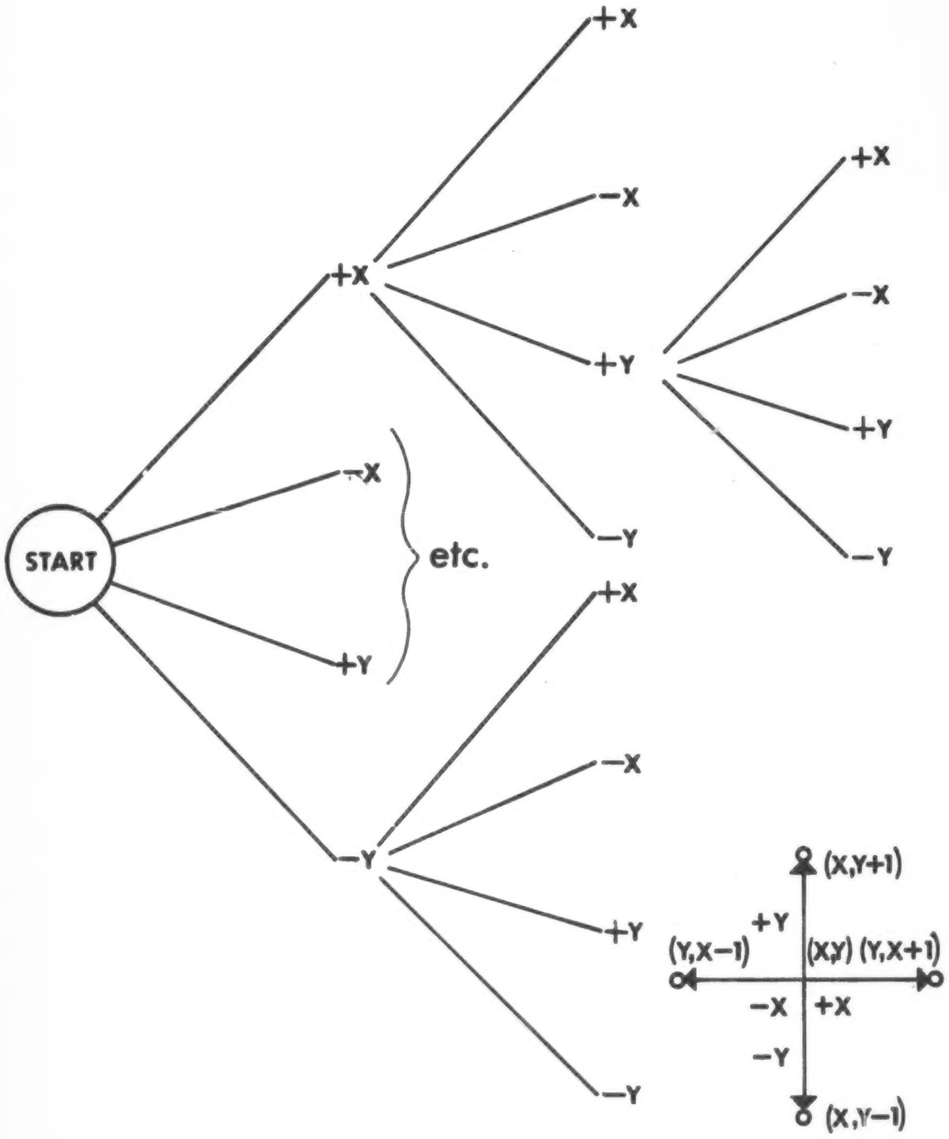

START
+X
-X
+Y
-Y
etc.
+X
-X
+Y
-Y
+X
-X
+Y
-Y
+X
-X
+Y
-Y
(X,Y+1)
+Y
(Y,X-1)
(X,Y)
(Y,X+1)
-X
+X
-Y
(X,Y-1)

Figure 2-3 Tree of Random Walk Moves

particular random walk described here can be solved analytically, but we can easily conjur up simple variations for which the analysis gets sticky:

a) There are two drunks moving randomly: given their starting positions how long does it take them to meet?

b) There are many particles uniformly spread in 3-space. (A random step can now also be up or down.) When two randomly walking particles meet they coalesce and a two particle system moves at half the speed of one, etc. After deciding what to do about collisions that do not occur at lattice points, we ask: What does the particle size distribution look like as $t \to \infty$?

In the last variation one is forced for the first time to consider events taking place in time. While it is true that the Buffon's Needle experiment, the game of "clock" and the simple random walk take place in time, the program is indifferent to whether the time period is seconds, hours or centuries. In the last variation of random walk, time is an intrinsic variable since now events are occuring at different rates and being initiated asynchronously. Time no longer passes in discrete steps of equal size. Thus, we may classify simulation programs as static or dynamic and the latter as synchronous or asynchronous. The simulation programs developed herein are dynamic, asynchronous models and we are now ready to describe the implementation and exercise of such a model:

> "... the problem or system under study is first described as the sequence of individual operations to be performed. [E.g., move truck from garage to first collection area, fill truck, go to disposal site, etc.] This may be called the 'Flow Model' of the system. It is then necessary to have data indicating how the individual operations are interrelated and to have the frequency distribution of elapsed time [and refuse production, work rates, etc.] for each individual operation for the different conditions to be explored.
>
> "Then inputs of such items as manpower, scheduling methods, or amounts of equipment, facilities, etc., are systematically varied. By consulting the time data mentioned above in a random manner [i.e., sampling from the frequency distributions], the [random] overall time for the sequence of operations can be determined. This process performed over and over simulates opera-

> tion of the system and permits accruing such total system data as average equipment and manpower utilization .... Such outputs are then used to evaluate the desirability of the given input under test, and in effect a simulated experiment has thus been conducted. [10][11]

Thus, the major problems for which Monte Carlo[12] simulation is used are those entailing complicated cascades of random events taking place (usually) in time and (usually) asynchronously. This is an apt description of a refuse collection system in operation.

Having dwelt at some length on the limitations of other methods of analysis, we would be remiss if we did not point out the weaknesses that are inherent in simulation.

First, simulation along with other methods of analysis has the problem of securing reasonably accurate data. When certain items of data are not available, one must either recast the analysis to avoid their use, secure the data or run the simulation with a reasonable range of values bracketing the supposed true value. But with simulation, the data problem is even more acute; one needs to know not only normative (average) values for certain parameters, but since many of these parameters are really random variables one needs to know the form of the associated probability density functions (or higher moments). A further difficulty one faces with data when preparing a simulation is the problem of the independence of random variables. To some extent this is taken care of in the flow model of the system, when one allows certain events to occur only when others have first occurred but this is not the whole story.

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[11] Italics added.

[12] So called due to the intimate connection existing between the calculus of probabilities and games of chance.

In the field of solid wastes one of the first places one confronts the data problem is in providing a refuse generation model. We have defined this above as being a rule or procedure for predicting the amount of refuse generated (random variable) given all the relevant factors. Present refuse generation models are crude, and this is one of the most active areas in the field of solid wastes today.

A second major limitation on simulation enters when the simulation entails human behavior. A point in the flow model may be reached where a decision or action is required by a human: how can we simulate the actual decision process? Some of the alternatives are:

a) Always do the same thing.

b) Provide a list of possible actions and the associated probabilities (based on observation) of their being invoked. Are we sure that we have observed enough cases to be aware of all alternatives?

c) Build some feedback or "learning" into the program so that successful responses are reinforced and unsuccessful ones punished or inhibited.

d) Try to simulate the human mind's process of scanning the present states of the system (including perhaps its propensity to err and to tire) to weigh the possibilities before acting. Here, one has replaced the one large decision by a host of smaller ones, and one may not have achieved anything thereby.

A refuse collection system is, of course, a system in which human decision processes enter into the final result. A prime example is the decision as to when a collection truck is sufficiently full to return to the disposal site. This problem will be discussed more fully in Chapter 3, where we discuss some of the assumptions and concepts that went into the present simulation programs.

A third major limitation on simulation is again one that is common to other methods of analysis. This is the problem of detail versus aggregation, of accuracy versus practical computation times, etc. Here, as elsewhere, the guiding principle has to be: what am I going to use the model for? The physicist builds a model of mechanics that assumes frictionless planes, weightless levers, etc; engineers must reintroduce the physical reality of friction. The navigator gets along with a Mercator projection, an analyst of satellite trajectories favors a globe while a cosmologist can get along with a point mass; all are models of the Earth. We recognized this principle above when we simulated Buffon's Needle by generating random numbers, not by considering the mechanical problem of a needle-shaped object falling through a viscous medium under the force of gravity. We recognized this when we simulated the game of "clock" by leaving the cards in place. So too in simulating the actions of refuse collectors there is detail which exists, but consideration of which may be neglected in order to answer the sort of questions for which the simulation model is intended. Thus, for example, we ignore such realities as coffee breaks and lunch hours.

A fourth and final limitation on the efficiency of simulation (and intimately related to the immediately preceding) is the set of restrictions imposed by the computer itself (speed, capacity), computer languages and the current state-of-the-art in simulation (which is recognized as a valid special interest area by the Association for Computing Machinery).

When all is said and done, one has in hand a program exhibiting, hopefully, the best advantages and avoiding the worst disadvantages. A necessary step ignored in the quotation above is the process of verifying the model. This means, for example, that when probability densities having mass = 1 concentrated at one point (the average) are used, results predictable in the absence of random variation should be achieved. Then when the necessary data are secured concerning a specific area, the answers generated should be in reasonable agreement with observation. Then one is ready to use the simulation model to extrapolate as suggested above. Every run must be regarded as a trial of validity and results clearly out of line with reason must bring either the model or the line of reasoning into question.

The number of problems, research programs, etc., using simulation techniques has multiplied in recent years far out of proportion to the growth in the number of computers on which to run them, hence it is neither possible nor desirable to provide a comprehensive bibliography on simulation. However, we do wish to mention three works which have had influence on the development of this simulation program. The first is simulation in social science: readings [sic] edited by Harold Guetzkow [10] which we have quoted above. The second work is Microanalysis of Socioeconomic Systems: A Simulation Study [11] which contains among other models a demographic model attempting to simulate the growth of populations from such elementary considerations as birth, death, marriage, propensity to remarry, etc. The third work, one not generally available, is a collection of mimeographed course notes from the University of Michigan [3].

Section C

The Simulation of Refuse Collection and Haul Operations

The desire to experiment with the use of Monte Carlo simulation for the analysis of refuse collection and haul operations grew out of an awareness of the successful use of the method in many other fields (as cited above), out of a study of the rational methods developed by the University of California [1], and finally out of an awareness that the day-to-day operation of a refuse collection and haul service is subject to many random variations which are glossed over by these rational methods. The rational methods use average refuse production for an entire city, average collection efficiency for all crews and distance of the center of the city to the disposal site to represent average one-way haul distance; this is macroanalysis. Simulation allows one to consider all of the inhomogeneities of time, place, and circumstance to whatever degree of picayune detail one can tolerate. Can such microanalysis yield anything that the macroanalysis cannot? As stated by Quon, *et al*:

> "The rational method has proven adequate for the purposes of providing an initial selection of

> equipment and operating conditions and an evaluation of the annual cost of the collection system, where basic data and experience pertaining to systems of a similar character are available."[13]

What the rational method cannot provide which simulation can is some indication of how the rationally planned system actually **performs** according to such criteria as length of workday, efficiencies of equipment utilization, etc. The averages may be fine, but there are real operational headaches involved in large fluctuations above or below the average. Moreover, experimentation in the field can yield adjustments that improve the rationally planned system, but the scope of this experimentation **is limited.** Thus, simulation may be regarded as a means for rapidly performing experimental changes on a system to delineate the interdependencies of the various parameters. Such results, combined with mature engineering judgment can select the most promising combination of changes. (Greatest desired change per dollar is one obvious **criterion.** It can be stated with some assurance that the parameters of primary importance are not the same in all places at all times.

The simulation programs reported here were based on the methods of collection of the Village of Winnetka, Illinois using data collected there in 1958 and on the City of Chicago. Among the kinds of results seemingly yielded by the simulation program in these circumstances is the idea that the daily route method[13] is unsuitable when the ratio σ/u (standard deviation/mean) for refuse production exceeds 0.15[14].

The significance of this particular result is large because (no matter the actual value) it gives a quantitative bound on the usefulness of a particular collection discipline where none existed before. A systematic management science for solid wastes can only be built out of such pieces.

At one time it would have been appropriate to extrapolate from the above to a speculation that simulation would be an important tool in the future of solid wastes. This is no longer necessary; the future has arrived and as we have noted, simulation is being used widely.

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[13] See Chapter 3.

[14] See Chapter 4.

CHAPTER 2

RATIONAL METHODS FOR DESIGN OF REFUSE COLLECTION/HAUL SYSTEMS

As stated in the previous chapter, the first comprehensive scientific effort to develop rational methods and design criteria for refuse collection/haul systems was developed by the University of California Sanitary Engineering Research Project in the early 1950's. The results of this work are reported in An Analysis of Refuse Collection and Sanitary Landfill Disposal , Technical Bulletin No. 8, Series 37 (December 1952)[15].

Because of the intrinsic importance of this work and because many of its concepts and results have been borrowed in the development of the present simulation program, it is necessary and proper to devote this chapter to a detailed exposition and discussion of the report. To accomplish this, the balance of this chapter will follow this outline:

A. An overview of the University of California Report.

B. Factors Influencing the Pickup and Haul

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[15] "The report" for the balance of this chapter.

Operations.

C. Development and Use of Rational Design Formulas.

D. An Evaluation of the University of California Report.

Section A

An Overview of the University of California Report

The motivation for the study itself and for its methodology is best expressed by the following quotations from the report:

> "The rational design of a refuse collection system is an engineering problem. The efficiency of a collection system depends on the judicious combination of the size of the collection vehicle, number of men per vehicle, the type of refuse pickup, and the number of trips per day for a given haul distance to the disposal site."
>
> "The principal reason for resolving refuse collection into the unit operations ...-pickup, haul, off-route, and at the disposal site - is to permit a rational analysis and design of the collection system."
>
> "An economic analysis of refuse collection systems entails the evaluation of the two principal components of cost: the labor cost and the operating expense."

An analytic scheme with such aims must eventually have numerical inputs and some knowledge of factors in the environment which influence these inputs. Accordingly, a field survey was conducted on thirteen California cities during 1950 and 1951. The results of this data gathering are reported in three tables for which a summary follows:

Table I: Presents unit refuse production in each of the thirteen cities in two volumetric and two gravimetric units: yds^3/service/week, ft^3/capita/day, lbs/service/week, and lbs/capita/day. Volumetric units are not used after their initial appearance here. This is because observations cover both open-body and mechanical-packer trucks, so that the compaction ratios have to be known in the case of the latter. These are hard to determine since first of all they depend on the composition and the moisture content of the refuse, and second inefficient use of the packers was observed to be common. The use of lbs/capita/day also vanishes, no doubt because in conducting a field survey it is easier to count the houses than the people on a block.

The refuse production figures are presented as city-wide averages for all trips observed within a given city. The cities are segregated into three groups on the basis of frequency of collection. The first group (eight cities) with primarily once-a-week collection showed an "average" refuse production of 32.3 lbs/service/week[16/] The second group with primarily twice-a-week collection (combined refuse in both groups) showed an average of 47.0 lbs/service/week. The third group represents two cities having separate collections of garbage (including commercial swill -- primarily restaurant wastes) and trash at different frequencies. The ratio of the group II to group I average is ~1.4. This propensity to set out more refuse when more frequent service is provided though not supported by tests of statistical validity has been observed by Quon and Tanaka [14] and more recently by Smallwood and Galler[17/] This figure, 1.4, is used in a curious computation on manpower requirement which will be enlarged on below.

[16/] Quotes have been used on the word average because this author is dubious of the way they are computed. Rates, percentages, etc. unless based on equal numbers of observations should be done by weighting, i.e. if p_i = refuse production (lbs/svc/wk) and S_i = number of services in the i-th city the average should be computed as

$$\overline{p} = \Sigma\, p_i\, s_i \div \Sigma\, s_i$$

not

$$\overline{p} = \frac{1}{n} \Sigma\, p_i$$

where the summation is over n cities. A similar criticism applies to many of the other averages displayed throughout the report. Having no detailed information from which to recompute them, we will continue to use them without further comment.

[17/] Personal communication.

Table II: Presents pickup time measured in man-minutes per ton for the commercial districts of seven cities. Additional information presented for each city includes crew size (men), commercial pickups as percent of total, and tons per trip. The average pickup time is reported as 136 man-min/ton.

Table III: Presents pickup time measured in man-minutes per ton for primarily residential areas of eleven cities (19 observations = 4 cities both 1950 and 1951 and 7 cities either 1950 or 1951). The additional information provided includes type of truck (packer or open), crew size, rear of house pickups as percent of total, residential pickups as percent of total, service density (services per mile), and a breakdown of the pickup operation by percentage of time spent in sub-tasks such as on truck, on property, loading, etc. The average residential pickup time is 148 man-min/ton. The discrepancy between the commercial and residential times can be ascribed chiefly to greater container density for the former.

The basic data presented, the report then turns to an analysis of factors affecting refuse production, pickup time and haul time. This part of the report will be discussed in greater detail in the next section.

The analytic models are based on the division of the total operation into the unit operations of pickup, haul, off-route, and at disposal site. These are defined as follows:

Pickup: Net time consumed during actual collection from the first stop with an empty truck to the last stop before proceeding to the disposal site.

Haul: Net time required for the round trip to the disposal site. This begins with the last container of a given route and ends either with arrival at the first container of the succeeding route or at the garage or city yard at day's end. It excludes time spent at the disposal site unloading or waiting to unload.

Off-route: A catchall for time spent resting, smoking, eating, on coffee breaks, contacting residents or supervisors, or not otherwise ascribable to the other three activities. This has been set at a flat 15 minutes per trip throughout the balance of the report. No allowance for this factor is made in the simulation programs.

At Disposal Site: Unloading or waiting to unload. This is set at a flat 5 minutes per trip for the rest of the report. It is treated as a random variable in the simulation with waiting time dependent on the number of trucks competing for the unloading spaces.

The total labor component of the cost is built out of these unit operations in the following word formula.

$$\begin{array}{c} \text{Total Man-Minutes} \\ \text{per Trip} \end{array} = \begin{array}{c} \text{Pickup} \\ \text{Time} \end{array} + \begin{array}{c} \text{Haul} \\ \text{Time} \end{array} + \begin{array}{c} \text{Off-route} \\ \text{Time} \end{array} + \begin{array}{c} \text{At Site} \\ \text{Time} \end{array}$$

One may divide this formula by average tons of refuse per trip to get a formula expressed in man-minutes per ton. The latter formula when multiplied by the average wage rate in dollars per man-minute yields the labor cost of the operation in dollars per ton.

The operating expense component of the cost is obtained from the formula

$$\begin{array}{c} \text{Total Operating} \\ \text{Cost per Year} \\ \text{per Truck} \end{array} = \begin{array}{c} \text{Annual} \\ \text{Depreciation} \\ \text{(Str.Line)} \end{array} + \begin{array}{c} \text{Average} \\ \text{Annual} \\ \text{Interest} \end{array} + \begin{array}{c} \text{Operation \&} \\ \text{Maintenance} \\ \text{per Year} \end{array}$$

This is divided by total tons collected per year per truck to yield operating expense in dollars per ton which is commensurate with the labor cost.

A final section of this report is devoted to using the formulas to design a system for a hypothetical city. We shall review this example in section C of this chapter.

Section B

Factors Influencing the Pickup and Haul Operations

A significant part of the report is concerned with establishing quantitative measures of the effect of various environmental and system design factors on refuse production, the pickup operation, and on the haul operation. In this section, we shall examine these findings and comment briefly on how they relate to the simulation programs.

We have noted the effect of frequency of collection on refuse production. A similar effect was noted on pickup time "other potential variables being constant". The ratio of pickup times for once and twice weekly collections was observed to be 1.2.

This is accounted for by noting that in each of the two twice-a-week collections the crew must cover the same amount of walking on the property and container handling as in the one once-a-week collection and all this for seven-tenths of the base amount of refuse. Using the 1.2 ratio stated the time requirement for the twice-a-week collection is .7(1.2) = .84 or 16% less than that for a once-a-week collection. (See the comment on the use of this ratio (1.2) in section D of this chapter.) A part of the field survey noted only cursorily above consisted of breaking the pickup operation down into component operations and observing the percent of time devoted to each. These component operations and the average percentages (Table III) are:

(a)	On Truck	24.9
(b)	On Street	15.5
(c)	On Property	17.6
(d)	At Container	14.1
(e)	Loading	20.1
(f)	Waiting	3.7
(g)	Resting	4.2

Time studies such as these can be useful when data are available to indicate how variations in the collection methods, equipment, etc. affect them. Then formulas may be devised to predict the change in overall pickup time which in turn can be plugged into the formulas for rational design, the development of which we shall trace in the next section. Or such predicted changes can be used as input to the simulation program.

The frequency ratios 1.4 and 1.2 which we have be been discussing do not enter into the simulation program except insofar as the user of the program uses them to alter the input data.

A second factor which was readily observable by the field survey was the effect of container location. Each pickup that was observed was classified as either a "rear-of-house" pickup or an "alley or curb" pickup, and the resultant trip was noted as to its percentage of the former. (Zero percent ROH = 100% AOC, etc.) Rear-of-house implies that the collectors must enter upon the property and either transfer the refuse to larger "tote" containers or bring the resident's containers out to the street and (in some areas) return them when emptied.[18/] Alley collection implies that there are alleys bisecting the blocks (going out of fashion) and the "on property", "at container", and "on street" components of the pickup are reduced, the latter because the alley makes both sides of

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[18/] In Redlands, California, rear-of-house collection is practiced with the collector driving a skip-loader tractor up the driveway to the container. He then empties the container into the hydraulically operated bucket on the front of his vehicle. The bucket is set at a convenient loading height. After several such visits, he finds the packer truck and uses the hydraulic system to empty his load directly into the compactor. More than one such tractor is assigned to a given truck. This system obviates some of the drawbacks of rear-of-house collection as well as some of the inefficiencies of mechanical packer trucks.

a block accessible simultaneously. Curb collection requires the resident to put his containers on the curb on the appointed day(s) and remove them when emptied. It is possible to spot two cities using 100% or nearly so alley or curb pickup in the data of Table III. These are Burbank and Long Beach with zero % of time in each spent on property or at container and on street components of only 5 and 6% respectively. The effect of container location on pickup time is shown by the trend line (least square?) of Chart 1. Zero percent ROH yields 104 man-min/ton on the line while 100% ROH yields 162 man-min/ton. (See remarks about this chart in section D of this chapter).

Container location *per se* does not enter into the simulation program except as it is employed by the program user to alter input data.

A third major influence on pickup time observed in the field survey was the use of open-body vs. mechanical packer trucks. The comparisons between truck types is made in Tables IV and V. While we have expressed some doubt about these comparisons in section D below, taking them at face value we find that open-body trucks are more efficient with an average pickup time of 135 man-min/ton while mechanical packers take 159 man-min/ton. In the analytic sections of the report, the discrepancy is taken to be 10%. The chief mechanism for this difference as discerned from Table V seems to be the waiting component of the pickup, i.e. time spent at the compactor with container to be emptied while the compactor digests material just put into it. For open-body trucks this comes to 1.9% of pickup time (or .019 × 135 = 2.57 man-min/ton) while for mechanical packers it is 7.0% of pickup time (or .07 × 159 = 11.13 man-min/ton for a net difference of 8.56 man-min/ton). Again, truck type influences the simulation program only via data input.

While the use of open-body trucks may be a moot point as we enter the 70's, this choice as well as frequency of collection and container location together with decisions on separation of refuse types and types acceptable for collection exemplify the general consideration "level of service expectation" which was discussed in the Preface.

Turning to the haul operation, the basic determinant is, of course, distance, but comparisons of efficiency can be made if the labor time for the haul expressed in man-minutes per ton is normalized by the distance to yield haul efficiency in man-minutes per ton-mile. The observations range from a low (best) of 1.75 man-min/ton-mi at Lodi for a 21.9 mile trip with an average 4.34 tons per trip to a high of 7.63 man-min/ton-mi at Oroville for a 7.8 mile trip with an average of 1.83 tons per trip.

The significant observation about the haul is that the average speed of the collection vehicles increases with the distance travelled (Chart 5) hence if a = crew size, c = tons carried per trip, T = time for trip in minutes, d = distance in miles and S = speed in miles per minute we have for E = haul efficiency.

$$E = \frac{a}{c}\frac{T}{d} = \frac{a\,T}{c\,ST} = \frac{a}{c\,S} \qquad (2.1)$$

or

$$\frac{dE}{dd} = \frac{-a}{CS^2}\frac{dS}{dd} < 0 \qquad (2.2)$$

since we have noted that $\frac{dS}{dd} > 0$. This is seen in Chart 4.

Haul efficiency is an output of the simulation and is as we shall see subject to wild distortion by small partial loads. Chart 5 is used by the simulation program to provide the mean speed of the refuse vehicles.

Section C

Development and Use of Rational Design Formulas

The word formulas which have been introduced above can be rendered quantitative by the introduction of the following variables. The symbols used are those given by the report itself and differ from those used in discussing the results of the simulation runs. In the use of the formulas, the report assumes, as noted, a flat 15 minute per trip off-route and 5 minutes per trip at dispoal site time.

a = Number of men per truck (crew size).
b = Pickup time in man-minutes per ton.
c = Average tons of refuse per trip.
d = Round-trip haul distance in miles.
e = Average (inverse) haul speed in minutes per mile.
f = Total man-minutes per trip.
g = Number of trips per day.
h = Initial cost of truck in dollars.
i = Interest rate on capital.
k = Total number of trips per year.
s = Useful life of truck in years.

m = Total mileage per trip = d + route mileage.

Ø = Operation and maintenance of truck, dollars per mile.

q = Total operating cost per truck per year.

x = Total man-minutes per ton.

y = Total operating cost (fixed charges, operation and maintenance) in dollars per ton of refuse collected.

Accordingly, the formula for total man-minutes per trip becomes

$$f = bc + ade + 15a + 5a \qquad (2.3)$$

where the components on the right are respectively pickup time, haul time, off-route time, and at disposal site time per trip. Dividing both sides by c we get

$$x = \frac{f}{c} = b + \frac{a}{c}(de + 20) \qquad (2.4)$$

total labor time in man-minutes per ton for all unit operations. Finally, if we let (these symbols are not used in report) w = Average wage rate in dollars per man-minute.

C_L = Total labor cost in dollars per ton

$$C_L = wx \tag{2.5}$$

which gives the total labor cost in dollars per ton.

Formula (2.4) is solved in the report by the construction of nomographs, Chart 6 for long hauls and Chart 7 for short hauls. Here, of course, we shall solve (2.4) directly. The proposed use of formulas (2.4) or (2.5) is to gauge the efficiency of existing systems. Thus, if we have

d = 16 miles.

e = 3 min/mile (20 mph).

a = 3 men.

c = 3 tons/trip.

b = 150 man-min/ton.

w = .025 \$/man-min (\$1.50/hr).

$$C_L = .025[150 + \frac{3}{3}(16 \bullet 3 + 20)] = .025(218) = \$5.45/\text{ton}$$

which may be compared to the existing situation by reducing the annual refuse budget by 10% (to account for administration and employee benefits), subtracting off the fixed and operation maintenance costs of the fleet, and dividing by total tonnage collected to get the comparison figure.

Operating costs per year per truck are obtained from the second formula of section B as follows:

$$q = \frac{h}{s} + \frac{hi}{2s}(s + 1) + km\phi \qquad (2.6)$$

where the three terms on the right are respectively annual straight line depreciation, average annual interest, and operation and maintenance per year per truck. If this is now divided by ck = total tons per year per truck we get

$$y = \frac{h}{cks}[1 + \frac{i}{2}(s + 1)] + \frac{m\phi}{c} \qquad (2.7)$$

the total operating cost in dollars per ton. In formula (2.7), the two terms on the right are respectively fixed charges and operation and maintenance.

Continuing the above example, if

m = 20 miles/trip.

g = 3 trips/day.

$h = \$2300.$

$s = 6$ years.

$\phi = .16$ \$/mile.

$i = .04.$

We have with the additional information that the trucks work 6 days per week. $k = 52 \cdot 6 \cdot 3 = 936$ trips/yr/truck and

$$y = \frac{2300}{3(936)(6)}\left[1 + \frac{.04}{2}(6 + 1)\right] + \frac{20(.16)}{3}$$

$$= \$.15/\text{ton} + \$1.07/\text{ton} = \$1.22/\text{ton}$$[19]

As before, nomographs are provided to facilitate solution (but restricted to $i = .04$).

Thus, the total collection cost for this system is \$6.67/ton exclusive of administration and employee benefits (\$5.45/ton + \$1.22/ton).

In considering formula 2.7 it should be noted that other methods for computing depreciation and average interest are sometimes used, that salvage value of the truck at the end of its useful life is ignored, that the factor ϕ itself consists of fixed and variable items, and that in considering operating expense on a per truck basis economies of scale are ignored. It is interesting also to speculate on whether formula (2.7) can be used to answer such a question as: How much more would it be worth to put into $\emptyset$ if doing so would purchase an extra year of useful life? Proceeding on the naive assumption that this is possible, one finds the break-even point to be 2-tenths of a cent per mile or 20 × 436 × .002 = $37.44 per truck per year. This seems too good a bargain to be true.

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[19] The report erroneously introduces \$1.82/ton into subsequent computations after first finding \$1.21/ton from the nomographs.

Turning from the evaluation of existing systems to the design of a total system from the ground up, and basing such designs on a fixed length of work day of 8 hours (480 minutes) and fixed number of trips per day we obtain from formula (2.3)

$$f = \frac{480a}{g} = bc + a(de + 20) \qquad (2.8)$$

or dividing by a

$$\frac{480}{g} = \frac{bc}{a} + de + 20 \ . \qquad (2.9)$$

Formula (2.9) is solved in four nomographs, Charts 10 - 13 for (g = 2 and 3) × (e = 3 and 3.5) .

The example used to illustrate the use of formula (2.9) is the following (slightly simplified). In the following the example we shall concentrate on the design selected as most practical and instead of tabulating all the inputs, we shall introduce them where first needed.

In performing this sample design, an eight-fold table (Table VI) is constructed: $(g = 2 \text{ and } 3) \times (a = 2 \text{ and } 3) \times$ (open-body and packer-body). Of these, the most economical proves to be impractical because it requires too large a load (tons/trip). The computations for the second best design are then

$b = 147$ man/min/ton.

$g = 2$ trips/day.

$a = 2$ men.

$d = 4$ miles round trip to incinerator.

$e = 3.5$ min/mile.

Determine c from (2.9)

$$\frac{147}{2} c = \frac{480}{2} - 20 - 4(3.5)$$

or $c = 2.80$ tons/trip, and from (2.4) and (2.9)

$$x = \frac{a}{c} \cdot \frac{480}{g} = \frac{2}{2.8} \cdot \frac{480}{2} = 171 \text{ man-min/ton} .$$

Thus, if $w = .025$ \$/man-min

$$C_L = .025(171) = \$4.28/\text{ton} \ (\$4.27 \text{ in report}) .$$

Finding the operation expense is not as simple as the report purports since the route mileage (I) which must be added to d to get m must be found:

Assuming that the truck works 6 days a week, 52 weeks a year, the total yearly tonnage for a truck is

$$\frac{6 \text{ days}}{\text{week}} \times \frac{2 \text{ trips}}{\text{day}} \times \frac{52 \text{ wks}}{\text{year}} \times \frac{2.8 \text{ tons}}{\text{trip.truck}} = \frac{1747 \text{ tons}}{\text{yr.truck}} .$$

But we know that residential refuse production is

$$12160 \text{ svcs} \times \frac{52 \text{ wks}}{\text{year}} \times \frac{1 \text{ ton}}{2000 \text{ ton}} \times \frac{48 \text{ lbs}}{\text{svc.wk}} = \frac{15175 \text{ tons}}{\text{year}}$$

Hence there must be $\dfrac{15175 \frac{\text{tons}}{\text{year}}}{1747 \frac{\text{tons}}{\text{yr.truck}}} = 9$ trucks[20/] in the fleet (for residential pickup).

Thus, with residential density at 90 svcs/mile and collection frequency at 2 trips/route/week

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[20/] (covered quotient)

$$I = \frac{12160\ \text{svcs}}{9\ \text{trucks}} \times \frac{2\ \text{trips/route/week}}{\frac{2\ \text{trips}}{\text{day truck}}\ \frac{6\ \text{days}}{\text{week}}} \times \frac{1}{\frac{90\ \text{svcs}}{\text{mile}}} = \frac{2.51\ \text{mi}}{\text{route}}\ .$$

Finally, $m = d + I = 4 + 2.51 = 6.51$ or 7 to the nearest mile.

In computing the operating expense we assume

$h = \$7500.$

$k = 624$ trips/yr$(= 6 \times 2 \times 52)$.

$i = .04.$

$s = 6$ years.

$\phi = .16$ \$/mile.

Thus,

$$y = \frac{7500}{2.8(624)(6)}(1.14) + \frac{7(.16)}{2.8} = \$1.20/\text{ton}.$$

Thus, assuming an incineration cost of \$2.15/ton we have a total cost of $4.28 + 1.20 + 2.15 = \$7.63$/ton (\$7.62/ton in report).

As a sample of the questions which this rational design method can answer, consider:

If sanitary landfill disposal costs \$.66/ton, how far can the refuse be hauled at the same total cost (exclusive of administration, etc.) as that just computed for incineration?

In order to answer this question and to eliminate the need for trial and error with the nomographs, we propose here to extend these rational methods by developing a formula which relates the economic capacity c to the round-trip haul distance. We shall assume

(1) 52 weeks operation per year.

(2) 20 minutes/trip off-route and at disposal site. (The place to change this in the formula is obvious.)

(3) e(the inverse speed, in min/mile) will be treated as a constant rather than as a function of distance.

(4) In computing the number of trucks we shall ignore the need for a covered quotient.

We shall need the following additional notation:

D = Days worked per week.

F = Shorthand for $\frac{1}{s}\,[1+\frac{1}{2}(s+1)]$.

f = Frequency of collection (trips/route/week).

J = Job, i.e. number of services.

K_1 = Total cost for collection, haul and disposal by incineration (\$/ton).

K_s = Cost for disposal (only) by sanitary landfill.

P = Refuse production (lbs/service/week).

ρ = Service density (services/mile).

T = Number of trucks in the collection fleet.

Repeating some of the computations for I in general form

$$\text{Total Yearly Tons per truck} = 52\ Dgc \tag{2.10}$$

$$\text{Total Yearly Refuse Production tons/year} = \frac{52\ JP}{2000} \tag{2.11}$$

$$T = \frac{(2.11)}{(2.10)} = \frac{JP}{2000\ Dgc} \tag{2.12}$$

$$I = \frac{J\delta}{TgD\rho} = \frac{2000\ \delta c}{P\rho} \tag{2.13}$$

Finally, noting that $k = 52\ Dg$

$$K_1 - K_s = C_L + y$$

or

$$K_1 - K_s = wb + \frac{wa}{c}(de + 20) + \frac{hF}{52\ Dgc} + \left[\frac{d + 2000\ \delta c}{P\rho}\right]\frac{\phi}{c}$$

whence

$$[K_1 - K_s - wb - \frac{2000\ \delta\phi}{P\rho}]\, c = [wae + \phi]d + 20\ wa + \frac{hF}{52\ Dg}\ . \tag{2.14a}$$

Inserting all known values except d and c and using $e = 3$ we have for $a = 3$, $g = 2$

$$3.137\ c = .385\ d + 3.784 \tag{2.14b}$$

Thus, if trucks of 4 tons capacity are a practical standard size (costing $7500), a haul of 22.8 miles (one way 11.4) would be practical. In fact, the method of trial and error yield $d = 18$, $c = 3.38$ which lies on the line (2.14b). Considering the compaction ratio this yields an odd-size truck which is rounded up to 15 yards.

Section D

An Evaluation of the University of California Report

The basic method for supplying scientific insight and predictive power in an area where little or none has been used before is to observe, to devise an unambiguous terminology to describe the phenomena, to define useful units for measuring those entities which can be measured, and to provide a conceptual framework for viewing the interrelations of component parts. In that this has been accomplished by the University of California report, it is praiseworthy. If there is anything of a major nature to criticize (besides the method of taking averages) it is the over-reliance on the use of averages as a means of comparing group characteristics without any statistical tests on the validity of those comparisons.

The data gathered in the field survey phase of the project fairly cry out for the use of analysis of variance for the separation of effects and the assessment of interactions. Thus, for example, in assessing the determinants of pickup time, such factors as service density, open vs. packer, percent alley-curb vs rear-of-house, crew size, city, and year data was gathered are all valid classifications for a nearly textbook analysis of variance model. This lack of statistical sophistication is reflected in Table IV where we are told that mechanical packers average 159 man-minutes per ton to 135 for open-body trucks despite a glaring difference of 33 containers per mile between the two groups. Again, in Chart 1 we note that of the 17 points plotted to give a trend line on the pickup time as a function of percent rear-of-house pickups, five of the six open-body points are above the line and seven of the eleven packer points are below the line; doesn't this justify the use of two lines?

A curiosity in the report is the following computation based on the factor 1.4 = ratio of average twice-a-week production to once-a-week production and the factor 1.2 = ratio of twice-a-week pickup time to once-a-week pickup time:

> "Thus, if three men were required to provide once weekly pickup service to a given area, theoretically 3×1.2 plus $3(1.4 - 1.0) = 3.6 + 1.2$ or 4.8 men would be required to supply the same area with twice weekly service."

The actual answer depends on the assumption one makes as to how much time it should take to perform the two collections: Assume base data of 1 ton refuse production from a given area and a pickup time of 150 man-min/ton collected once-weekly.

1. Require that the two collections be done in the same time (T_1) as the singleton collection

$$T_1 = \frac{150 \frac{\text{Mm}}{\text{T}} \times 1\text{T}}{3\text{M}} = 50 \text{ min}$$

$$\therefore M_2^{(1.0)} = \frac{180 \frac{\text{Mm}}{\text{T}} \times 1.4\text{T}}{50\text{m}} = 5.04 \text{ men } (= 1.2 \times 1.4 \times 3) .$$

2. Decide that pickup time is proportional to the amount of refuse and therefore require the job to be done in $1.4T_1$ minutes

$$M_2^{(1.4)} = \frac{180 \frac{\text{Mm}}{\text{T}} \times 1.4\text{T}}{70\text{m}} = 3.6 \text{ men } (= 1.2 \times 3) .$$

3. Decide that the "terrain" is the main influence on pickup time hence require the job to be done in $2T_1$ minutes

$$M^{(2.0)} = \frac{180\ \frac{Mm}{T} \times 1.47}{100m} = 2.52 \text{ men } (= \tfrac{1}{2} \cdot 5.04) \ .$$

Each of these is correct under the assumptions stated. The question is which is the correct assumption? This author believes that the correct assumption is a mixture between the extremes of 2. and 3., although as we shall see below, 3. is more nearly correct for the Village of Winnetka, i.e., the physical requirements of the collection procedure (walking, toting, etc.) dominate refuse production as the determinant of pickup time.

It should be pointed out also that the unit operations defined here do not fit a situation in which the collection personnel do not accompany the driver and truck to the disposal site. One such collection discipline, the relay or shuttle method, as described by Refuse Collection Practice [5 - pp.153-154] has a driver with an empty truck meet the collectors at the point where the previous truck left off. A modified relay method is evaluated in Chapter 5.

CHAPTER 3

DEVELOPMENT OF THE SIMULATION PROGRAM

Section A

Sources of Variability in Refuse Collection/Haul Operations

In the Preface, we noted that the significant parameters of a refuse collection system are subject to variability or fluctuation. We identified these types of variation as being long-term or secular variations, seasonal or cyclical variations, and short-term random fluctuations. This work concentrates on the latter two types of variation, and to these we add, by way of contrast to the use of average values only, the real inhomogeneities of physical placement, i.e.,distances travelled by collection vehicles are measured from actual points of departure for the disposal site. We add, also, variable amounts of waiting time at the disposal site.

Moreover, in using simulation to study the effects of such variability on the operation of a refuse system we need, as we indicated in Chapter 1 of Part 2, knowledge of the probability laws governing the observed randomness. The use of random number generators to produce random inputs according to some probability law is concealed in every crevice of the simulation program. It is, therefore, the purpose of this section to reveal these places, the probability laws used, and the justification for their use.

Determination of Refuse Amounts: A survey of the Village of Winnetka in 1958 provided data which when plotted on arithmetic probability (graph) paper by individual trucks showed that the probability distribution of refuse production is reasonably close to normal. One such plot for truck PW43 is shown as Figure 2.4. For all five trucks, the mean (μ), standard deviation (σ), and number of trips in the sample (n) are shown in Table 2.1. The same data revealed, to a high degree of statistical confidence (Chi-square test), that in Winnetka's twice-a-week collection the refuse amounts produced when four days had elapsed since last collection were 4/3 the amounts produced when three days had elapsed. This is another way of saying that refuse production appears to be uniform over the refuse collection cycle. A further major source of variation in refuse production is seasonal variation. In most areas the increased availability of fresh produce in the summer months combined with lawn and garden trimmings raises the refuse

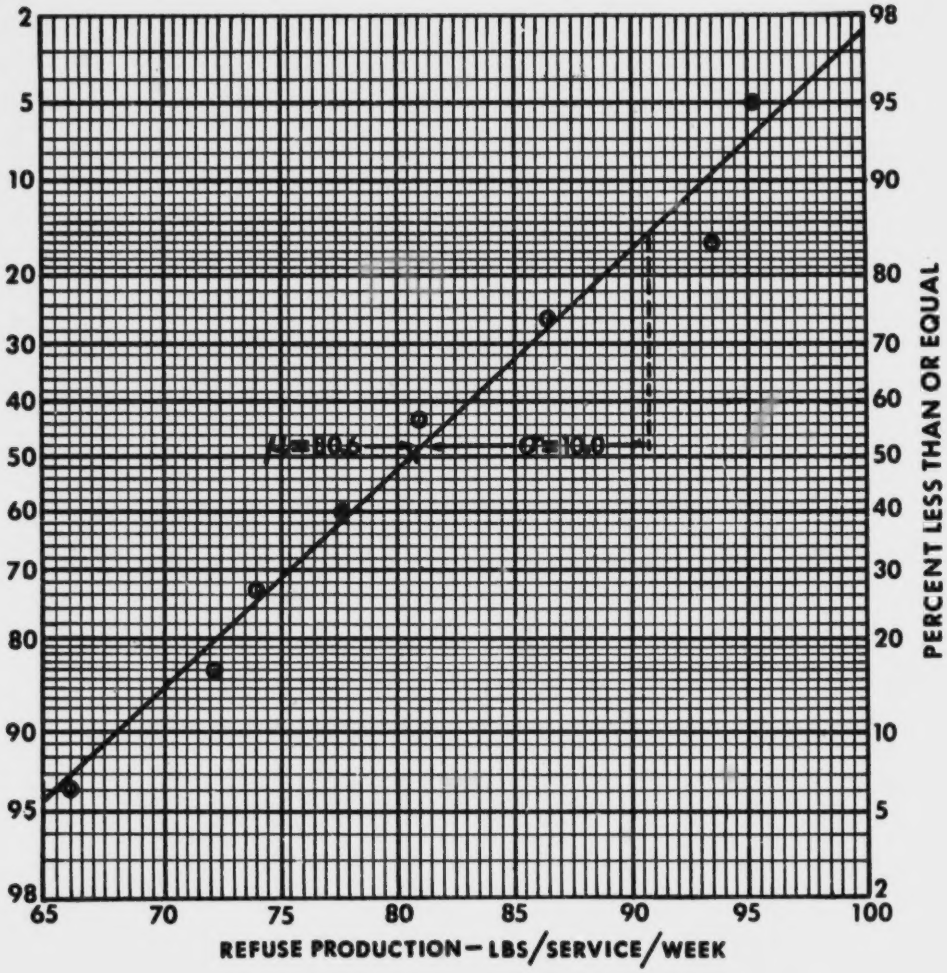

Figure 2-4 Probability Plot of Refuse Production, Winnetka, Illinois, March - April, 1958

Truck No.	Mean(μ)	Std.Dev.(σ)	σ/μ	Number of Trips (N)
PW 3	78.5	13.1	0.167	7
PW 12	75.1	19.9	0.265	8
PW 43	80.6	10.0	0.124	8
PW 47	84.0	5.6	0.067	7
PW 54	76.1	15.3	0.201	7

Table 2-1 Statistics on Refuse Production.

Winnetka, Illinois, March-April, 1958.

production of these months above that of the winter months. An example of this can be seen in Refuse Collection Practice [5] in Figure 14 (p. 34) which shows the weekly production in tons for a full year in Cincinnati, Ohio. The magnitude of the summer peaks is in the neighborhood of 1.3-1.5 times the winter and spring levels. Provision for the influence of seasonal variation has been made in the simulation program. The detailed description of the "refuse-generator" is included in Appendix 2-A.

Determination of Pickup Time: Linear regression methods were used to relate the time of refuse collection (minutes) to the amount of refuse collected and the number of services. It was found that for Winnetka the effect of the amount of refuse is statistically insignificant so that the refuse term is dropped from the regression formula. The explanation for this is simply that almost all residential refuse in Winnetka is collected from the rear of the house in a city virtually without alleys. Thus the subcomponents of the pickup operation such as "on property" and "at container" are dominant in the pickup operation and much the same time is required no matter what the amount of refuse.

The resultant regression formula takes the form

$$\bar{\theta} = \text{Time (minutes)} = \alpha S + \gamma \qquad (3.1)$$

where S = number of services and

α, γ are the regression coefficients

This formula has been modified to include the effect of route mileage.(See Appendix 2-A.)

Common sense decrees that any activity conducted out-of-doors must be affected by the weather. A means for incorporating weather severity as a factor in the time computation is described in Appendix 2-A.

Determination of Distance: Each point in the collection area which could conceivably be the departure point for a trip to the disposal site (only one disposal site) is known by its physical coordinates on the city map. Thus, when a truck is full (see next section) the distance is computed from where it actually is. The method for distance computation is by the ℓ_1-or metropolitan metric which has been described in Part I . If the amount of refuse collected is a random variable then it follows that the point at which a truck of given capacity is fully loaded is also a random variable.

Determination of Haul Speed: The basic model for the determination of haul speed as a function of haul distance was derived from Chart 5 of the condensation of [1] appearing in Refuse Collection Practice [5] (Figure 29 of original report). The detailed description of this model is contained in Appendix 2-A. The haul speed derived from this model is used as the mean of a normal distribution with a standard deviation of 0.1 × the mean speed. There is no empirical justification for imposing random variation on haul speed in this manner except that it appears reasonable and is preferable to no randomness whatever.

Time Spent at Disposal Site: The disposal site was modeled after the incinerator plant of Evanston, Illinois. Here, there are a limited number of unloading docks which are used by the trucks on a first-come-first-served basis. Thus, a variable wait-to-unload time may be generated. This time may be eliminated simply by setting the number of unloading docks equal to or greater than the number of trucks.

Once an unloading dock has been secured, the actual unloading time is generated as a random variable uniformly distributed between 5 and 9 minutes. The choice of these limits and of the uniform (rectangular) density was based on conversations with Winnetka officials. For the simulation model reported in Chapter 2-5, based on the City of Chicago, the limits used were 5 to 15 minutes and 2 to 6 minutes.

Determination of Capacity: For the simulation program reported in Chapter 4, based on the daily route method (see section B below) of the Village of Winnetka, randomness was used as a method for simulating the behavior of the crews in determining when the truck was sufficiently full to require a trip to the disposal site. The capacity rule (see subroutine CAPRUL in Appendix 2-A) used was briefly:

1) If this was the last CU (see section B below) of the day, permit up to 5% overload.
2) If this was not the last CU of the day, generate a random pseudo-capacity = Truck capacity (tons) times a random variable from the normal density N(.95, .05). Measure this pseudo-capacity against refuse already loaded plus the refuse in the CU.

To quote from an early report on this simulation program:

"The motivation for 'pseudo-capacity' is that it

> reflects the actual behavior of refuse collecters we've observed more closely than would insisting that the truck be filled to capacity each time. Refuse collectors don't stop, for example, when half a container has been loaded onto the truck or when they are in the middle of a street block. They have a 'feel' for when the truck is getting full (usually determined by the way the truck handles or 'lugs') and stop collecting when they've reached a 'good cut-off point'." [16]

It will be observed, nonetheless, that the results reported in Chapter 2-4 exhibit the effects of small partial loads which are not prevented by this capacity rule. An alternative capacity rule applicable to the daily route method, gained by the wisdom of hindsight, will be discussed in Chapter 2-6. A look-ahead rationale is used in the constant length workday simulation of the City of Chicago reported in Chapter 2-5. This will be discussed in the next section under differences between the two programs.

Section B

Some Concepts Underlying the Simulation Programs

As has been noted above, there are two simulation models under discussion. The first is based on the daily route method of refuse collection as practiced in the Village of Winnetka, Illinois. The second model was produced by making modifications to the first and is based on the constant length workday method of refuse collection as practiced in the City of Chicago.

In this section we shall present some of the concepts and organizational methods that underline both models. Detailed information on the first model and on the common portions of the second may be found in Appendix 2-A. The specific differences in concept and organization incorporated into the second model will be discussed here.

Collection Units: The fundamental organizational concept underlying both models is the collection unit (CU henceforth). A collection unit is, ideally, the smallest, compact piece of collection territory serviced by a single truck for which certain data is available. It represents a compromise between the homogenizing aggregation of the city as a whole (e.g. the rational methods) and the cumbersome detail of considering the city service by service. The input information required for each CU are:

1) Average refuse production in pounds per service per day (μ).
2) The standard deviation of same (σ).
3) The regression coefficients α and γ mentioned in section A above.
4) The days of the week the CU is to be serviced.
5) The number of services (S).
6) The service density in services per mile (ρ_s). Internal mileage m_i is gotten from dividing services by service density.
7) Physical location: (x, y) coordinates in miles.

In actual practice, items 1) - 3) are properties of up to 10 "districts" which need not be physically compact, and these properties are conveyed to the CU's by stating to which district they belong. One may note that the data provided are based on observation. One might have provided more fundamental data on income, percent alley pickup, etc. and created a refuse generating program to predict refuse production based on these fundamental determinants.

The important property required in designating an area to be a CU is that errors arising from considering it to be homogeneous be negligible. The meaning of homogeneity, insofar as the simulation models are concerned, is that a CU is proportionally divisible. Thus, if a truck can only load $\frac{1}{3}$ of the refuse found in a CU, when it returns (as it must under the daily route method) the remaining $\frac{2}{3}$ of the refuse will be collected in $\frac{2}{3}$ of the time and the truck will travel $\frac{2}{3}$ of the miles (m_1) originally computed for the whole CU. Moreover, the location of the CU ((x, y) coordinates) apply to all such operationally defined subdivisions of the CU for purposes of computing distances to and from the disposal site or garage. Both simulation models assume that the string of CU's which comprise the day's work for the truck which services them are physically contiguous so that neither time nor mileage are expended in going from a completed CU to the next CU on the given day's schedule. A total of 1000 CU's is allowed.

Disposal Site and Garage: A single disposal site and garage are provided for. They are specified by their physical locations (which need not be the same) in a coordinate system consistent with that used to locate the CU's. In addition, the number of unloading docks (berths, platforms, etc.) available at the disposal site must be specified.

Trucks: Trucks are specified as to their number (up to 64 allowed), the crew size K , and capacity in tons (C).

Time Organization: The time organization of both models is based on the World Calendar quarter of 91 days (13 weeks). All summary data are printed out at the end of a quarter's operation. The next largest time unit is the month. A World Calendar quarter consists of three months of 31, 30, and 30 days in that order.[21] Weather induced slowdowns in work rate (man-minutes per ton) and seasonally related adjustments to refuse production are changed on a monthly basis. The customary seven day week is the basis for scheduling the work of the trucks. A given CU may be serviced by its truck up to six times per week. The next time unit is the day which is the time unit during which simulation takes place. Within a day are variable length entities called events.

---

[21] The 365-th and 366-th days (when required) are placed between December 30 and January 1. These are called World Day 1 and World Day 2 and do not correspond to one of the 7 days of an ordinary week. Thus, each quarter begins on Sunday and each year's calendar is the same as another's except for the number of World Days.

Program Organization: The actual simulation proceeds on a daily basis. This daily simulation is called the inner program. The inner program is governed by an outer program which regulates the counting of days into weeks, months, and quarters, and which tallies daily results into quarterly counters and prints results at the end of each quarter.

The day begins with the dispatching of each truck that is to work on a given day of the week to its first CU of the day and ends when all trucks have returned to the garage. In between lies a variable sequence of events of minimal length zero. These events (listed in the order of their implementation as computer programs) are:

1) Truck arrives at disposal site: Here a truck may encounter delay if no unloading dock is open. Once an unloading dock is secured, a variable random unloading time is incurred.
2) CU is entered for first time: The refuse amount, time and travel are computed and the capacity rule is invoked to determine whether the entire amount of refuse can be loaded.
3) Truck leaves disposal site: For the garage or to resume collecting.

4) Collection unit is partially completed: This event is created when event type 1 or 5 (below) determine that there is too much refuse in a CU to fit the remaining capacity of the truck.

5) Truck enters partially collected CU: This event is created when event type 1 or a previous event type 5 has occurred without finishing the CU.

6) Truck arrives at garage: The day ends when all working trucks perform this event.

Full program details and interactions of these events may be obtained from Appendix 2-A.

Within the inner program is an event scheduler called the event clock or the time status record (TSR). The function of the TSR is as follows: When events are created, they are set to occur at a certain number of minutes from the time of creation. At the same time, all pending events are stacked in a list telling how many minutes from now they will occur. The TSR takes the newly created event and places it in the list so that all these times are in ascending order. Thus, the earliest occurring (next) event is on the top of the list. When an event is to be executed, this top-of-the-list event is removed and the number of minutes from now when it was to have occurred is subtracted from the occurrence time of the other pending events. This same time is added to the time of the day counter -- thus, time passes in variable length segments -- asynchronously. For full details see Appendix 2-A.

Daily Workflow of a Single Truck: The simulation models are best understood in terms of a workflow diagram, of a single truck's operation for one day. It must be pointed out that in such a workflow diagram, the workings of the TSR and the actual program operation are lost.

The first workflow diagram pertains to the daily route method of refuse collection as practiced in Winnetka. Under this method, each crew is assigned one or more work areas which are to be serviced at some prescribed weekly frequency. The area assigned for any given day must be completely serviced on that day. Each crew operates independently, helped by and helping no other crew and quitting early or late as the workload demands. The workflow for this method of collection is shown in Figure 2-5.

Before considering the daily workflow of a truck operating under the constant length workday rules of

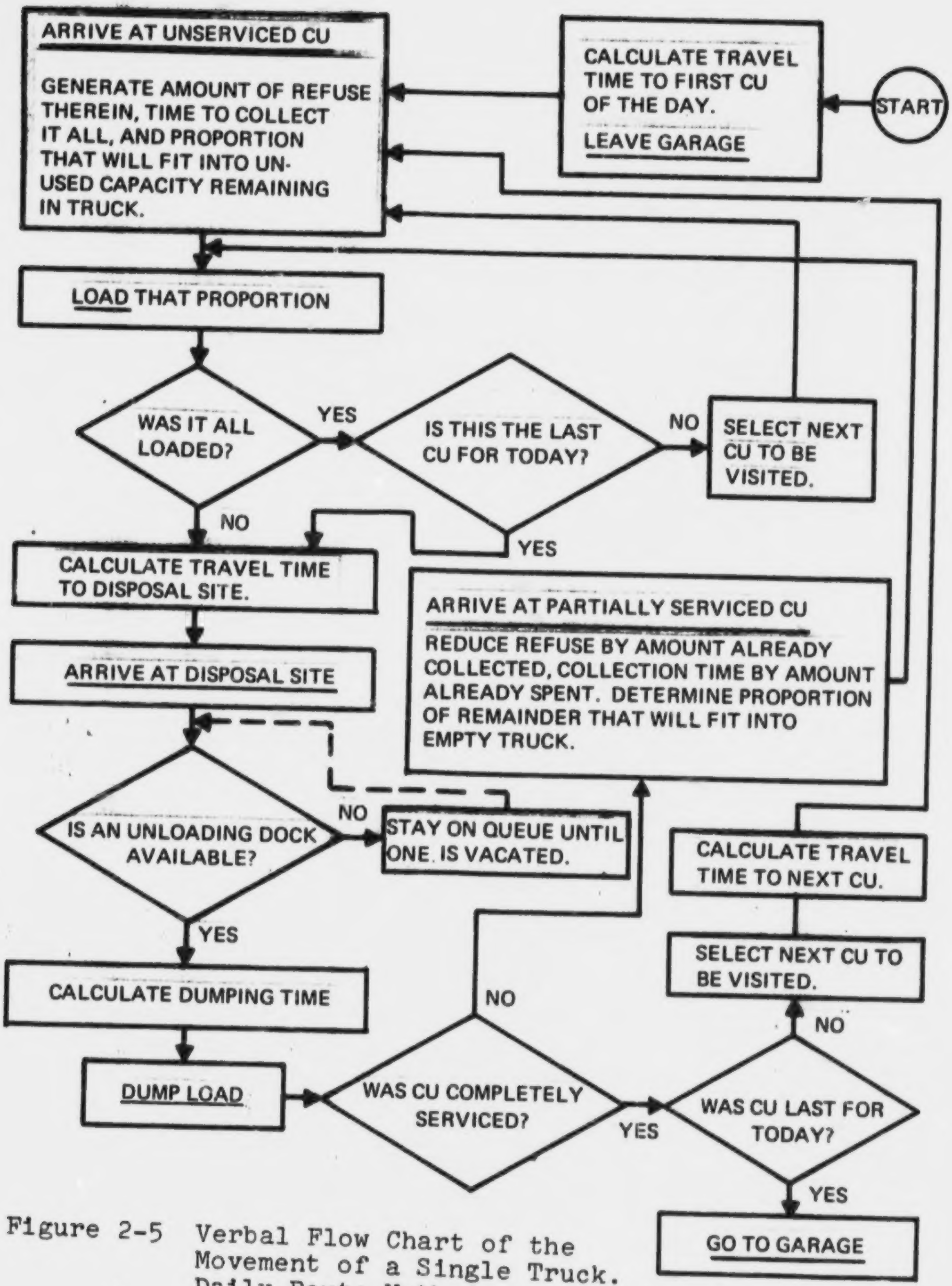
START
CALCULATE TRAVEL TIME TO FIRST CU OF THE DAY.
LEAVE GARAGE
ARRIVE AT UNSERVICED CU
GENERATE AMOUNT OF REFUSE THEREIN, TIME TO COLLECT IT ALL, AND PROPORTION THAT WILL FIT INTO UNUSED CAPACITY REMAINING IN TRUCK.
LOAD THAT PROPORTION
WAS IT ALL LOADED?
YES
IS THIS THE LAST CU FOR TODAY?
NO
SELECT NEXT CU TO BE VISITED.
NO
YES
CALCULATE TRAVEL TIME TO DISPOSAL SITE.
ARRIVE AT DISPOSAL SITE
ARRIVE AT PARTIALLY SERVICED CU
REDUCE REFUSE BY AMOUNT ALREADY COLLECTED, COLLECTION TIME BY AMOUNT ALREADY SPENT. DETERMINE PROPORTION OF REMAINDER THAT WILL FIT INTO EMPTY TRUCK.
IS AN UNLOADING DOCK AVAILABLE?
NO
STAY ON QUEUE UNTIL ONE IS VACATED.
YES
CALCULATE TRAVEL TIME TO NEXT CU.
SELECT NEXT CU TO BE VISITED.
CALCULATE DUMPING TIME
NO
NO
DUMP LOAD
WAS CU COMPLETELY SERVICED?
YES
WAS CU LAST FOR TODAY?
YES
GO TO GARAGE

Figure 2-5 Verbal Flow Chart of the Movement of a Single Truck. Daily Route Method, Winnetka, Illinois

Chicago, we must introduce some new terminology and spell out the ways in which this second model differs from the first.

The Constant Length Workday Model: In contrast to the daily route method, crews working on a constant length workday may leave over for servicing tomorrow any work which they are unable to finish today within the allotted workday. Any such work which is left over and then is not done the next day may be counted as not done. Thus, percentages of services serviced on time, one day late, and two or more days late provide an index to the quality of service being afforded. A model based on the constant length workday must have built into it a set of decision criteria simulating human judgment in answering the question: "Is there enough time left before quitting time for us to undertake the next task?"

Since this model is designed also to evaluate a last load relay policy, and since the use or non-use of this policy affects the way the criteria are computed, we first define this relay policy. The last load relay policy is simply to have the driver and the crew return directly to the garage with the last load of the day. From there, an auxillary driver takes the loaded truck to the disposal site, the regular driver and crew being allowed to quit within the allotted workday. This policy has, of course, cost implications in the wages of the auxillary driver, but these will be discussed in Chapter 2-5.

Following the verbal flowchart of the operation of a single truck under constant length workday rules, Figure 2-6 , the first place where such a criterion is examined is just before beginning to service a fresh CU. The question asked is: "Is estimated loading time available greater than CLT 1?" (constant length time 1). That is, if we take the number of minutes presently left until quitting time and if we subtract from this the average amount of time we shall have to spend traveling and the average amount of time spent unloading, will there be enough time (CLT 1) left to make it worthwhile to start servicing this CU? To express this as a formula, assume that we are using an 8 hour workday (480 minutes) and that

T = time elapsed up to now.

$\bar{T}_{CUD}$ = average time to travel from this CU to the disposal site.

$\bar{T}_{DG}$ = average time to travel from the disposal site to the garage.

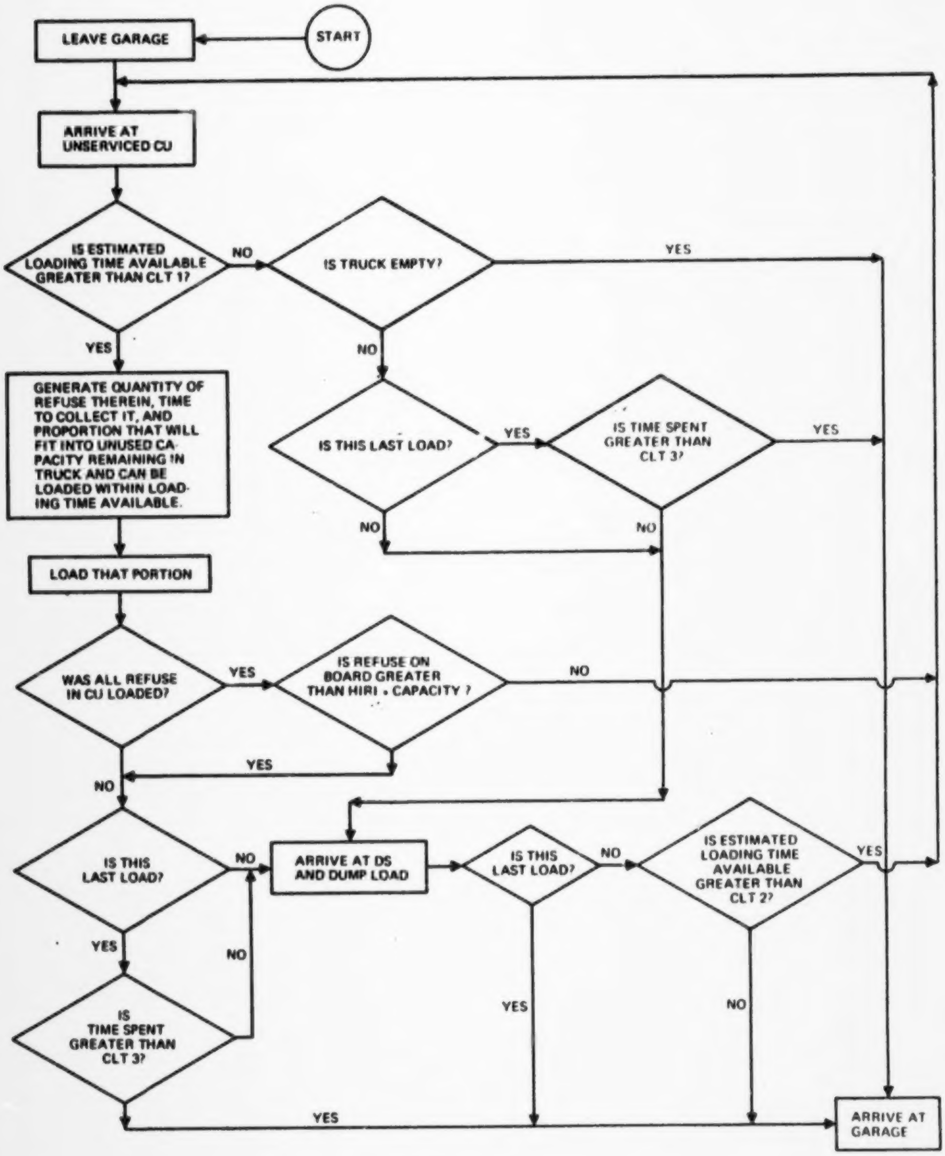
LEAVE GARAGE
START
ARRIVE AT UNSERVICED CU
IS ESTIMATED LOADING TIME AVAILABLE GREATER THAN CLT 1?
NO
IS TRUCK EMPTY?
YES
YES
NO
GENERATE QUANTITY OF REFUSE THEREIN, TIME TO COLLECT IT, AND PROPORTION THAT WILL FIT INTO UNUSED CAPACITY REMAINING IN TRUCK AND CAN BE LOADED WITHIN LOADING TIME AVAILABLE.
IS THIS LAST LOAD?
YES
IS TIME SPENT GREATER THAN CLT 3?
YES
NO
NO
LOAD THAT PORTION
WAS ALL REFUSE IN CU LOADED?
YES
IS REFUSE ON BOARD GREATER THAN HIRI • CAPACITY ?
NO
YES
NO
IS THIS LAST LOAD?
NO
ARRIVE AT DS AND DUMP LOAD
IS THIS LAST LOAD?
NO
IS ESTIMATED LOADING TIME AVAILABLE GREATER THAN CLT 2?
YES
YES
NO
IS TIME SPENT GREATER THAN CLT 3?
YES
NO
YES
ARRIVE AT GARAGE

Figure 2-6 Verbal Flowchart of the Movement of a Single Truck.
Constant Length Workday, Chicago, Illinois

$\overline{T}_{CUG}$ = average time to travel from this CU to the garage.

$\overline{T}_{DS}$ = average time spent at the disposal site.

$\overline{TV}$ = estimated time for non-loading activities.

ETL = estimated time left for loading.

Then for the non-relay case and for all but the last load of the day under the relay policy

$$\overline{TV} = \overline{T}_{CUD} + \overline{T}_{DG} + \overline{T}_{DS} \qquad (3.2a)$$

while for the last load of the day under relay

$$\overline{TV} = \overline{T}_{CUG} \qquad (3.2b)$$

Using whichever of these definitions of $\overline{TV}$ is appropriate

$$ETL = 480 - T - \overline{TV} \qquad (3.3)$$

and ETL is to be tested against CLT 1. In the examples discussed in Chapter 2-5, the value of CLT 1 = 15 min. and $\overline{T}_{DS}$ = 24 min.

The second place where this sort of look-ahead criterion is employed is at the disposal site when it is known that the previous load was not the last load of the day. The question, "Is estimated loading time available greater than CLT 2?", translates as "Is there **enough** time left to make it worthwhile going after the next load?"

For the non-relay case and for all but the last load of the day under relay

$$\overline{TV} = 2\ \overline{T}_{CUD} + \overline{T}_{DG} + \overline{T}_{DS} \qquad (3.4a)$$

while for the last load of the day under relay

$$\overline{TV} = \overline{T}_{CUD} + \overline{T}_{CUG}\ . \qquad (3.4b)$$

In the former of these two formulas for $\overline{TV}$, $\overline{T}_{CUD}$ is the average time to travel from the disposal site to the first CU of the next load (which may have already been started). Since we do not know, in general whether this is the same CU that will be being serviced at the end of the next load, we use $2\ \overline{T}_{CUD}$ as an approximation for the round trip time. ETL is computed as before, but CLT 2 is computed as a function of $\overline{TV}$:

$$\text{CLT 2} = \delta + \varepsilon \cdot \overline{TV} \qquad (3.5)$$

where δ = 20 min. and ε = 0.2.

Thus, the truck and crew return to an unserviced or incompletely serviced CU only when the net estimated time left for loading is greater than 20 minutes plus 20% of the estimated time for other activities.

The criterion CLT 3, queried when it has been determined that a non-empty truck is on its last load of the day, governs the use of relay. A value of CLT 3 greater than 500 means relay is not being used while a value of zero implies the use of relay for the last load of the day.

It should be noted that the number of trips per day is fixed in this model whereas there was no such limit in the daily route model.

Further differences between the two models include the capacity rule and the introduction of randomness into the refuse pickup rate (man-minutes per ton) in a different manner. The capacity rule used in the constant length workday model is simply to test refuse already on board the truck against an input parameter, HIRI, times the maximum capacity. Thus, for capacity at 5.5 tons and

HIRI = 0.9, a load of 4.94 tons would allow further loading to proceed, but only as far as 5.5 tons.

In the daily route model, the daily weather severity, WESEV, was derived as a random multiplicative constant from a month-dependent histogram. When $\bar{\theta}$, the pickup time for S services had been derived from the formula (3.1).

$$\bar{\theta} = \alpha S + \gamma$$

the actual value was found from

$$\theta = \bar{\theta} \cdot \mathrm{WESEV} \qquad (3.6)$$

In the constant length workday model, θ is found by

$$\theta = \bar{\theta} \cdot U(0.8, 1.2) \qquad (3.7)$$

where $U(0.8, 1.2)$ is a random variable uniformly distributed between 0.8 and 1.2.

CHAPTER 4

Simulation Studies of Refuse Collection and Haul under the Daily Route Method

The aim of the studies to be presented below is to delineate the interdependencies of several significant parameters involved in the functioning of a refuse collection system. The method of study is the use of a Monte Carlo or mathematical simulation technique implemented through a computer program. Monte Carlo techniques in general and this simulation program in particular have been discussed in Chapter 2 and 3 and Appendix 2-A.

We adopt the following point of view in undertaking these studies. First, we suppose that we have applied the rational methods discussed in Chapter 2 to the solution of the design problem of a situation akin to that of the Village of Winnetka. Next, we note that at many points where we made use of average in this design process, there is actually considerable day-to-day fluctuation and seasonal variation. We decide therefore, to study the effects of such variation in our initial design, but in doing so, we are not willing to explore these effects on too many variations of the base design. That is, we have exercised some mature engineering judgment in deciding that certain parts of the system, for example -- crew size, shall remain invariant. We are, thus, using the simulation program for a set of limited, economical experiments in order to discover without actual field trials both the effects of variability and which changes indicated by these effects will be most amenable or promising for field trial. We are not relying upon the results of these simulation studies to provide exact answers to questions arising from the everyday operation of a refuse collection system. We believe, rather, that the results from this method of analysis can be relied upon to describe the interaction of the variables involved and to differentiate between variables of primary and secondary importance.

The outline for the balance of this chapter is as follows: In section A we shall discuss the variables which we have held invariant in these studies. In section B we shall present the ranges of the variables allowed to change. In section C we present the results of some twenty-five simulation runs (each one run twice) through a series of graphs and tables. These results are interpreted, insofar as possible, on a physical basis. The conclusions reached apply, of course, only to the reference system. These conclusions may be made applicable to other similar systems by running the program on data applicable to those systems.

Section A

Invariants of the Reference Systems

Invariant parameters are, as we have noted, dictated by initial judgments on the relative importance of each relevant parameter; that is, the formulation of a tractable but meaningful problem entails decisions as to which variables to hold fixed and which to vary. The invariance of other parameters is dictated by dependency relationships between these parameters to output parameters or to a particular computational algorithm employed at the point or points where these variables impact on the system. Thus, for example, crew size is an invariant parameter if we do not (as we have not) build into the simulation program a rule relating crew size to pickup efficiency in a non-linear manner. That is, if two men collect refuse at 120 man-minutes per ton, and we assume through our computational rules that three men would collect the same refuse in 2/3 of the time, then pickup time is dependent on crew-size in a trivial linear manner and we gain nothing in varying both the pickup time and the crew size unless we pair the variations. The point to keep in mind is that simulation cannot generate new information; it is simply a method for making quick and economical computations of a complex nature on available information.

To summarize, the parameters held invariant are:

a) the daily route method discussed in Chapter 3;

b) crew size of two men, including the driver;

c) twice weekly collection frequency with each truck working four days a week, i.e., 52 days in a 13 week quarter;

d) no unloading dock bottleneck time at the disposal site.

Section B

Variable Parameters of the Reference System

The input parameters which are allowed to vary are summarized in Table 2-2. (A glossary of the symbols used to represent these parameters as well as the invariants and the various results or output parameters are given as an appendix to this chapter.) The values in Table 2-2 which are underlined are those which represent the base system. Consideration of Table 2-2 shows that the Cartesian product of the variables would result in 1388 different combinations. Rather than run all these cases many of which would be either ridiculous or superfluous and which would in any case require too much computer time and require excessive time for digestion and analysis, we have adopted a method of cross-sectioning in which only one variable at a time is varied with all other variables held at the base (underlined) values. The exceptions to this are μ, the refuse production in pounds per service per day, which is run vs. all the entabled values of the coefficient of variability σ/μ; and the truck capacity C (tons) which is also run against all values of the coefficient of variability. The resulting twenty-five cases are summarized on the bottom of Table

Input Parameter	Values			
Number of Services per Collection Unit, S	200	350	500	
Service Density, ρ_s, services/mile	35	70	100	
Haul Distance, m_h, miles	2	4	8	12
Quantity of Refuse, μ, lbs/service/day	5	10	15	
Coefficient of Variability, σ/μ	0.1	0.3	0.6	
Pickup Time Coefficient, α, min/service	0.5	0.7	1.0	1.3
Truck Capacity, C, tons	3	4	5	

Table 2-2 Value of Input Parameters

2-5 . (Note that A is not one of the input variables; A will be defined and discussed in section C.)

Closely related to the selection criteria for selecting these twenty-five cases is the decision to run the system in an uncoupled, single season manner; that is, as presently constituted, the only place where there is interaction between trucks is when they are competing for unloading docks at the disposal site, and it was therefore decided to treat the study of such competition as a separate special case. The other twenty-five cases are run as single truck runs. Moreover, for these twenty-five cases, the problem of seasonal fluctuation in refuse production has been handled not by running through a year using the seasonal factor mechanism of the model, but instead separate single production level quarters have been run with different levels of refuse production, number of services, etc. The study of the variability of work rate has not been handled via the weather severity mechanism of the model, but instead by running different quarters with changes in α the time per service coefficient discussed in Chapter 3.

The queuing problem at the disposal site, i.e., the waiting time at the disposal site as a function of the number of unloading docks provided, was studied for a fleet of ten 3.2 ton, two 4 ton, and one 2.5 ton truck (thirteen trucks). The first eleven trucks had 3 man crews, the last 2 men. The garage was located at coordinate (in miles) (5, 5) ; the disposal site at (10, 10) ; and the CU's were centered at regular points in a grid having both x and y increments of 0.2 miles with most CU's containing 600 services at a density of 60 services per mile (served once a week). A few CU's had as many as 1200 services at a density of 90 services per mile and were served twice a week. Refuse production variability ranged from σ/μ = 0.125 to 0.500 and the pickup rate varied between 98 and 300 man-minutes per ton with most crews collecting at 120 man-minutes per ton.

Section C

Results and Analysis

The results generated by the simulation program are reported on a quarterly basis, truck by truck and pertain to all the work assigned to that truck in that quarter. These results are divided into three major categories: Time Distribution, Crew Performance, and Truck Utilization. Additionally, frequency histograms are generated for the length of workday (H_w) and for truck capacity utilization (ℓ). A digest of the simulation results for the base cases 7 and 7R (repeat run) is shown in Table 2-3. The results shown in this table demonstrate the consistency of the results from run to run although it might be argued that this consistency results from the low coefficient of variability of refuse production $\sigma/\mu = 0.1$. An inspection of cases 9 and 9R which differ from cases 7 and 7R only in having $\sigma/\mu = 0.6$ shows maximum discrepancies of 8% in the average column. The discrepancies in the minimum and maximum columns are, as one would expect, larger and these will be apparent from the graphs of the results which appear below.

	Average		Minimum		Maximum		Standard Deviation	
Time Distribution								
total pickup time, ΣT_p, %	77.2	77.2						
total haul time, ΣT_h, %	18.5	18.5						
total time at disposal site, ΣT_d, %	4.3	4.3						
pickup time per trip, t_p, min	124	124	15.0	24.1	186	190	37.2	37.8
haul time per day, T_h, min	74.8	74.7	55.0	53.0	95.0	96.2	15.0	15.5
time at disposal site per trip, t_d, min	7.00	6.95	5.00	5.01	9.00	8.86	1.16	1.15
Crew Performance								
hours worked per day, H_w, hrs	6.72	6.72						
pickup efficiency, E_p, man-min/ton	101	100						
overall collection efficiency, E, man-min/ton	131	130						
haul efficiency, E_h, man-min/ton-mile	3.46	3.42	1.95	2.06	24.16	12.95	2.29	1.79
Truck Utilization								
total trips in quarter ΣN	52	52						
trips per day, N	2.50	2.50	2	2	3	3		
haul speed, V, mph	16.0	16.1						
effective truck capacity, C_e, tons	2.47	2.48						
refuse collected per day, L, tons	6.17	6.19	4.70	4.72	7.71	8.07	0.94	0.92
refuse collected per trip, l, tons	2.47	2.48	0.29	0.51	3.12	3.19	0.61	0.64
internal distance traveled per trip, m_1, miles	4.00	4.00	0.47	0.78	5.91	6.12	1.20	1.22

Table 2-3 Example Output Information from Simulation

(Time Span: 13 weeks, Runs 7 and 7R)

The frequency histogram results are shown in Tables 2-4a, 2-4b , and 2-4c. The capacity utilization histogram is gathered into twelve percentile groups (originally four percentile groups) and the results are shown in percent of trips rather than in absolute numbers of trips. These will be discussed at greater length below.

The results, other than the frequency histograms, are exhibited as a series of graphs, Figure 2-7 through 2-16. A concise presentation of results is obtained by confining our attention to three main parameters as the abscissae of these graphs. These are σ/μ , the coefficient of variability of refuse production; A (called A_r in Chapter 5, **the refuse assignment**, i.e., the average quantity of refuse assigned to a crew per day, expressed in truck loads. A may be computed by the following formula for twice-a-week collection

$$A = \frac{L_{avg.}}{C} \quad \frac{3.5\ \mu\ S}{2000\ C} \ . \tag{4.1}$$

Cases Included* → Assignment →	4	12	20	17	7,10,11 14-16, 23-25	13	1
↓ Percent of Capacity	1.02	1.17	1.23	1.53	2.04‡	2.92	3.06
$0 \le E_c < 12\%$	3.2	2.4	4.5	0.5	0.3	0.	0.
$12 \le E_c < 24\%$	16.8	4.2	5.7	1.4	1.2	0.8	0.5
$24 \le E_c < 36\%$	12.9	10.1	4.0	9.1	4.5	2.2	1.1
$36 \le E_c < 48\%$	0.	18.4	12.5	12.0	7.0	6.6	2.5
$48 \le E_c < 60\%$	0.	3.0	13.6	2.9	5.3	9.1	3.6
$60 \le E_c < 72\%$	0.	0.	0.6	6.3	6.9	5.2	5.5
$72 \le E_c < 84\%$	8.4	0.	1.7	10.1	8.1	3.9	11.8
$84 \le E_c < 96\%$	41.9	30.4	21.0	35.6	39.7	42.3	46.9
$96 \le E_c < 108\%$	16.8	30.9	36.4	21.6	26.8	29.7	27.2
$108 \le E_c \le 120\%$	0.	0.6	0.	0.5	0.2	0.3	0.6

* All cases include repeat run.

‡ No statistically significant difference was found in this group before summation. See Table 2.4c

Table 2-4a Distribution of Percent of Trips vs. Percent Capacity Used.

Coefficient of Variability $\sigma/\mu = 0.1$

Case* →	5		21		18		8		2	
Assignment →		6		22		19		9		3
↓ Percent of Capacity	1.02		1.23		1.53		2.04		3.06	
$0 \leq E_c < 12\%$	1.4	2.6	2.7	1.2	0.	2.4	0.4	0.4	1.1	0.3
$12 \leq E_c < 24\%$	10.0	10.3	5.5	8.2	5.2	4.4	4.1	4.6	3.4	4.1
$24 \leq E_c < 36\%$	6.8	7.7	8.2	5.9	7.6	3.4	3.3	6.2	2.6	3.0
$36 \leq E_c < 48\%$	6.7	8.4	6.0	5.9	5.7	7.3	4.5	5.8	3.7	2.7
$48 \leq E_c < 60\%$	6.1	9.0	8.2	8.2	4.8	8.3	7.4	4.6	3.4	3.3
$60 \leq E_c < 72\%$	2.7	7.1	8.2	5.9	6.7	5.9	3.0	5.4	3.2	4.6
$72 \leq E_c < 84\%$	14.2	7.1	7.1	10.0	8.6	9.8	7.1	4.2	4.8	5.2
$84 \leq E_c < 96\%$	34.5	25.2	32.2	32.3	38.1	31.7	39.3	41.9	47.0	42.1
$96 \leq E_c < 108\%$	17.6	22.6	21.9	22.4	22.8	26.8	30.9	26.9	30.8	34.7
$108 \leq E_c < 120\%$	0.	0.	0.	0.	0.5	0.	0.	0.	0.	0.

* All cases include repeat run.

Table 2-4b Distribution of Percent of Trips Vs. Percent Capacity Used. Coefficient of Variability $\sigma/\mu = \boxed{0.3 / 0.6}$

Group*	7	10	11	14	15	16	23	24	25	$N_{i\cdot}$
0 - 36%	11	16	13	13	16	21	18	16	17	141
36 - 48%	19	23	11	23	20	14	15	20	19	164
48 - 60%	13	12	24	13	13	8	16	13	12	124
60 - 72%	21	26	18	15	14	18	17	13	19	161
72 - 84%	17	16	22	14	22	27	23	32	17	190
84 - 96%	117	109	103	115	99	90	99	91	108	931
96 - 120%	62	60	69	67	77	82	73	76	68	634
$N_{\cdot j}$	260	262	260	260	261	260	261	261	260	2345 = $N_{\cdot\cdot}$

Table 2-4c Contingency Table Analysis of E_c Frequency

Histogram — A = 2.04, σ/μ = 0.1

Legend for Table 2.4c

Notes: 1) First three and last two groups of Tables 2.4a and b are **lumped** to assure each $n_{ij} \geq 5$.

2) Null hypothesis: H_o: For each row there exists a p_i such that $\Sigma_i p_i = 1$ and ∋ the probability of a result belonging to the i-th group = p_i for all cases.

3) Define $\chi^2(\text{sample}) = \Sigma_i \Sigma_j [(N_{ij} - \hat{p}_i N_{\cdot j})^2 / \hat{p}_i N_{\cdot j}]$ where $\hat{p}_i = N_{i\cdot}/N_{\cdot\cdot}$ (maximum liklihood estimate of p_i), and $N_{i\cdot}$ = row sum, $N_{\cdot j}$ = column sum, $N_{\cdot\cdot}$ = **grand total.**

4) $\chi^2(\text{sample}) = 55.71$ for (9-1)(7-1) = 48 degrees of freedom and $P_r(\chi^2 > \chi^2(\text{sample})] = 0.2074$.

5) $\chi^2(\text{sample})$ thus falls far to the left of χ^2_α for any reasonable region of rejection of H_o. ($P_r[\chi^2 > \chi^2_\alpha] = .05$ is usual.) Therefore, we accept H_o .

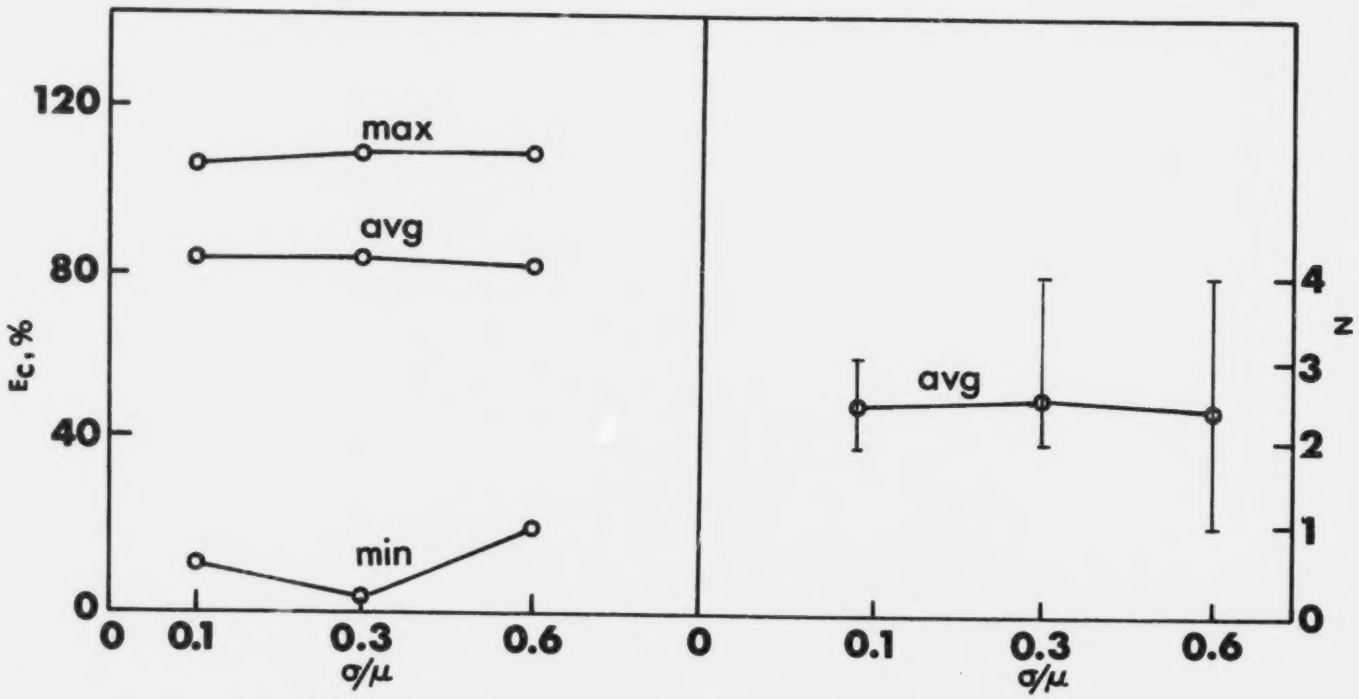

Figure 2-7 Truck Capacity Utilization and the Number of Trips per Day vs. the Coefficient of Variability

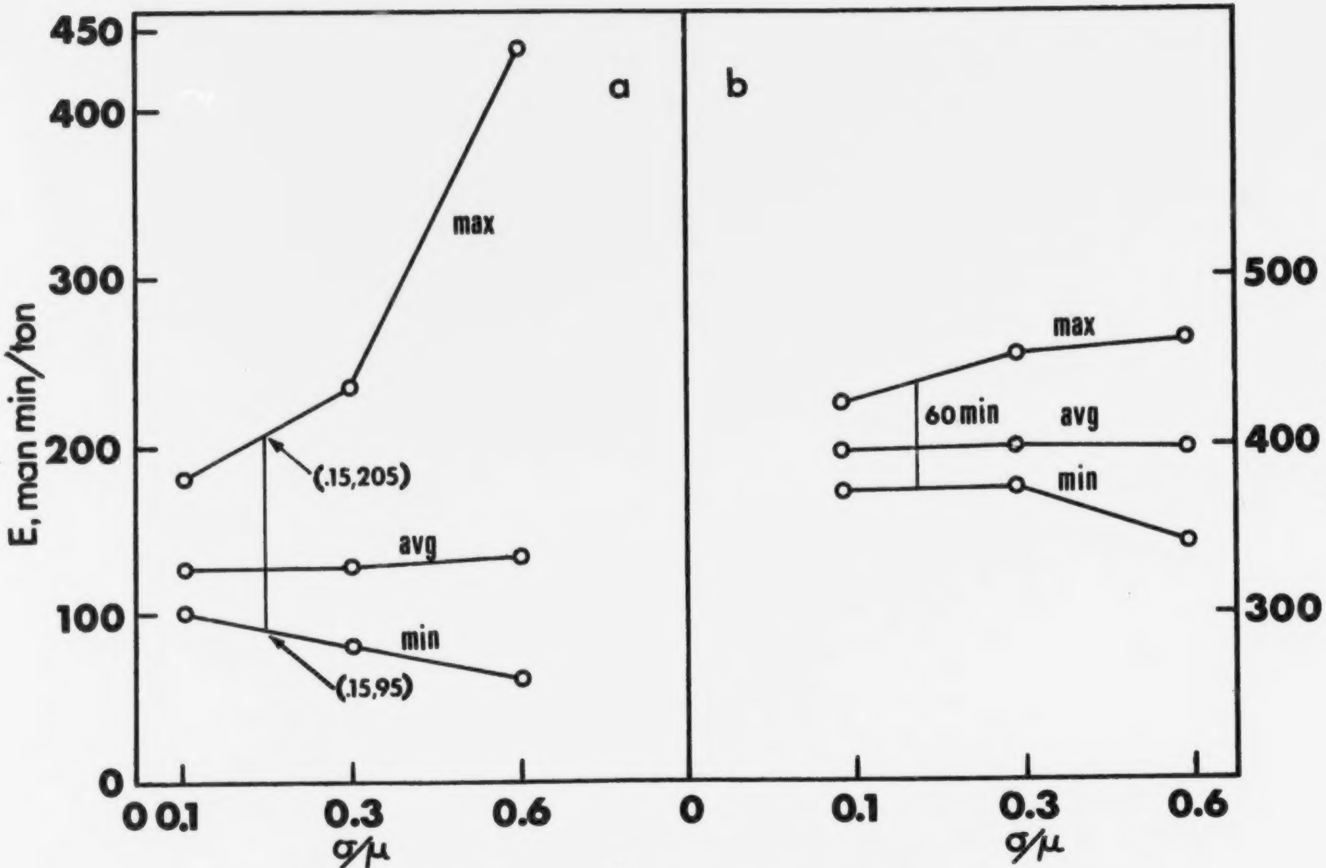

Figure 2-8 Overall Collection Efficiency and Length of Workday vs. the Coefficient of Variability

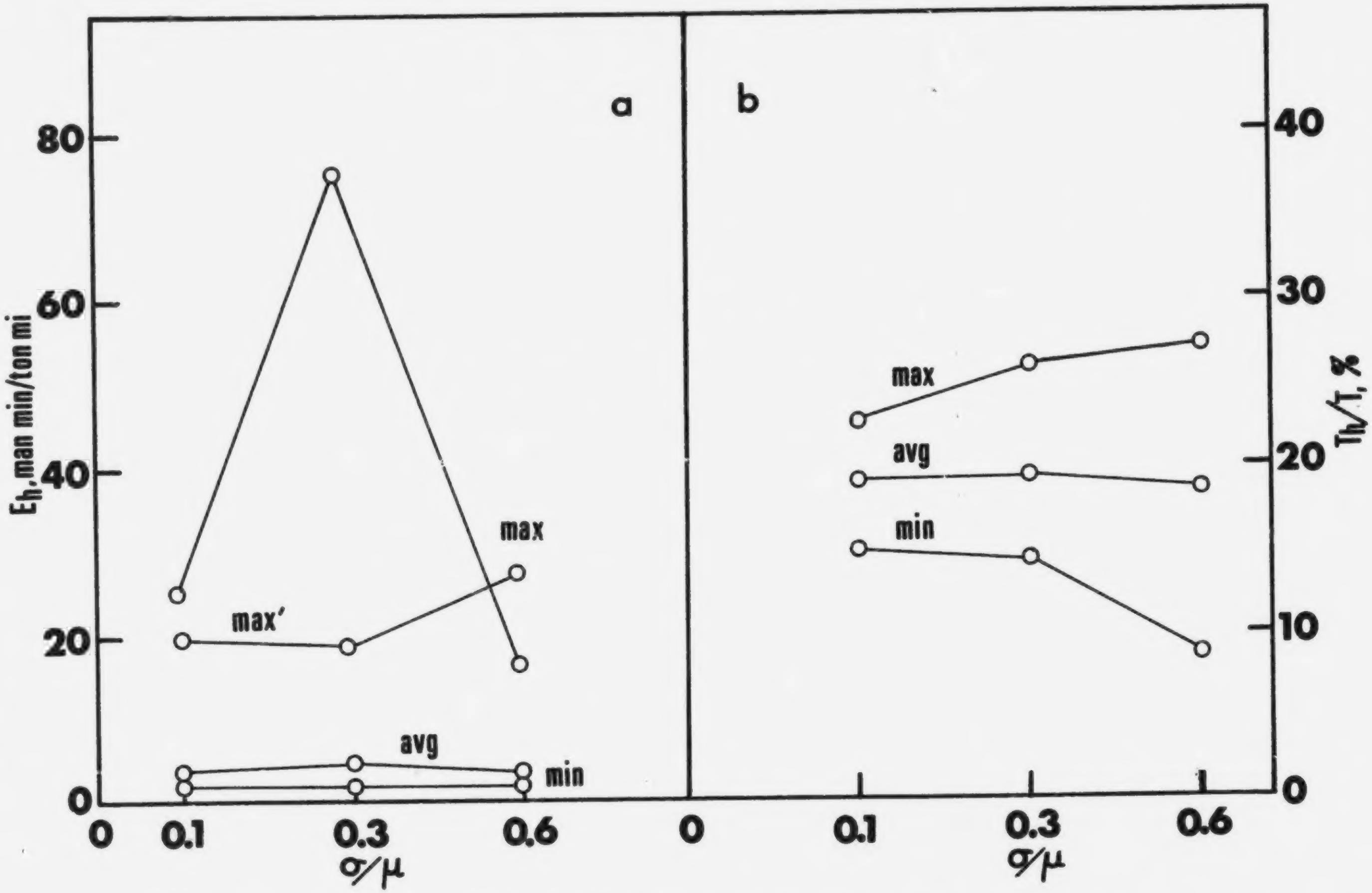

Figure 2-9 Haul Efficiency and Haul Time vs. the Coefficient of Variability

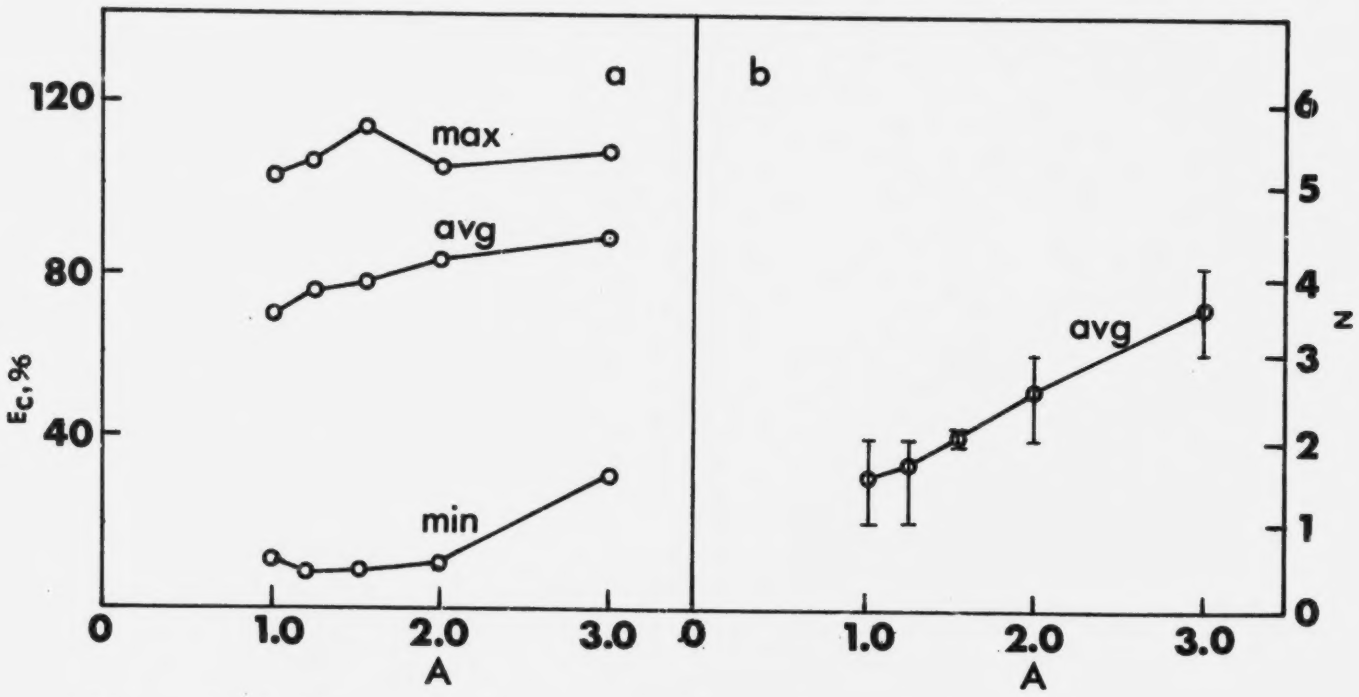

Figure 2-10 Truck Capacity Utilization and the Number of Trips per day vs. the Assignment

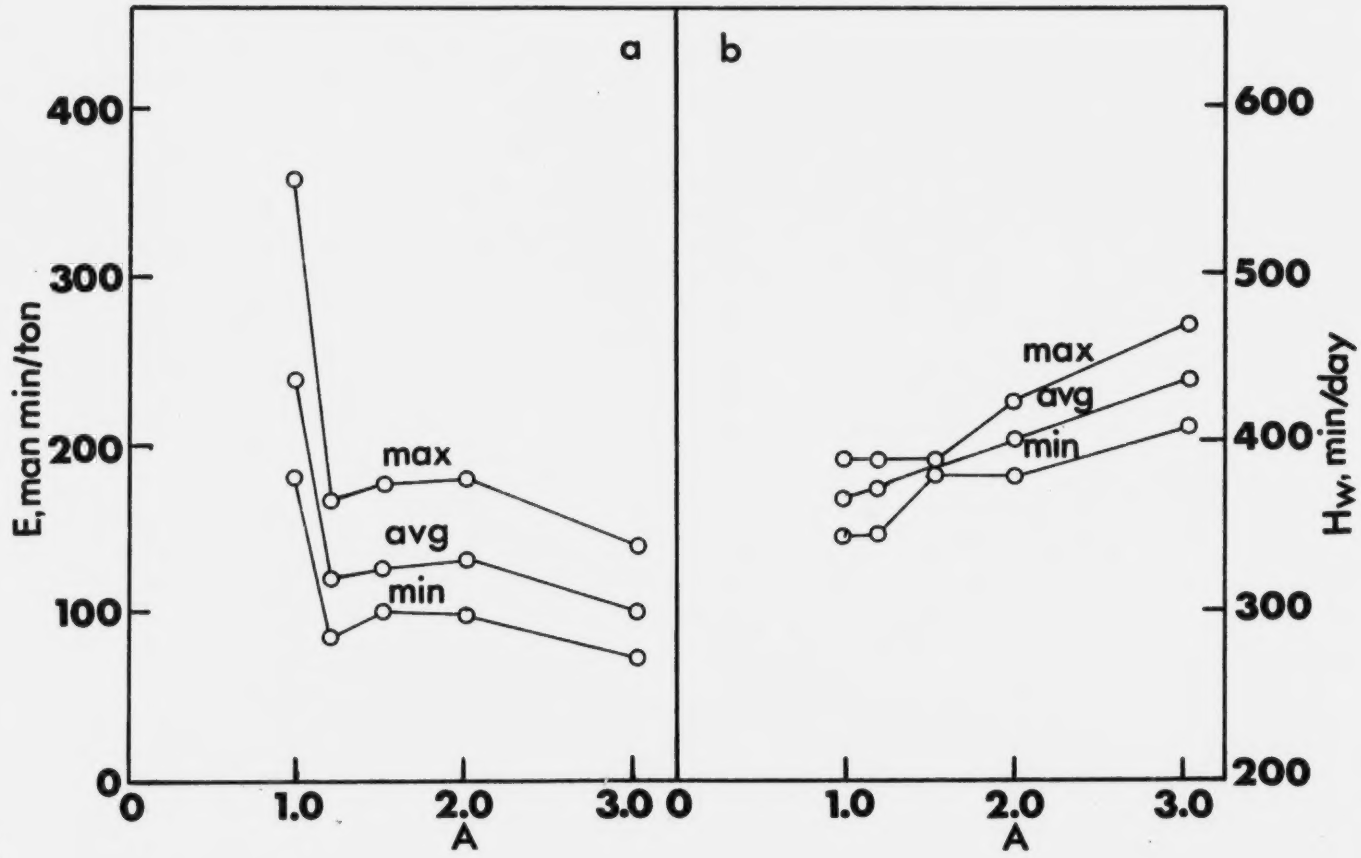

Figure 2-11 Overall Collection Efficiency and Length of Workday vs. the Assignment

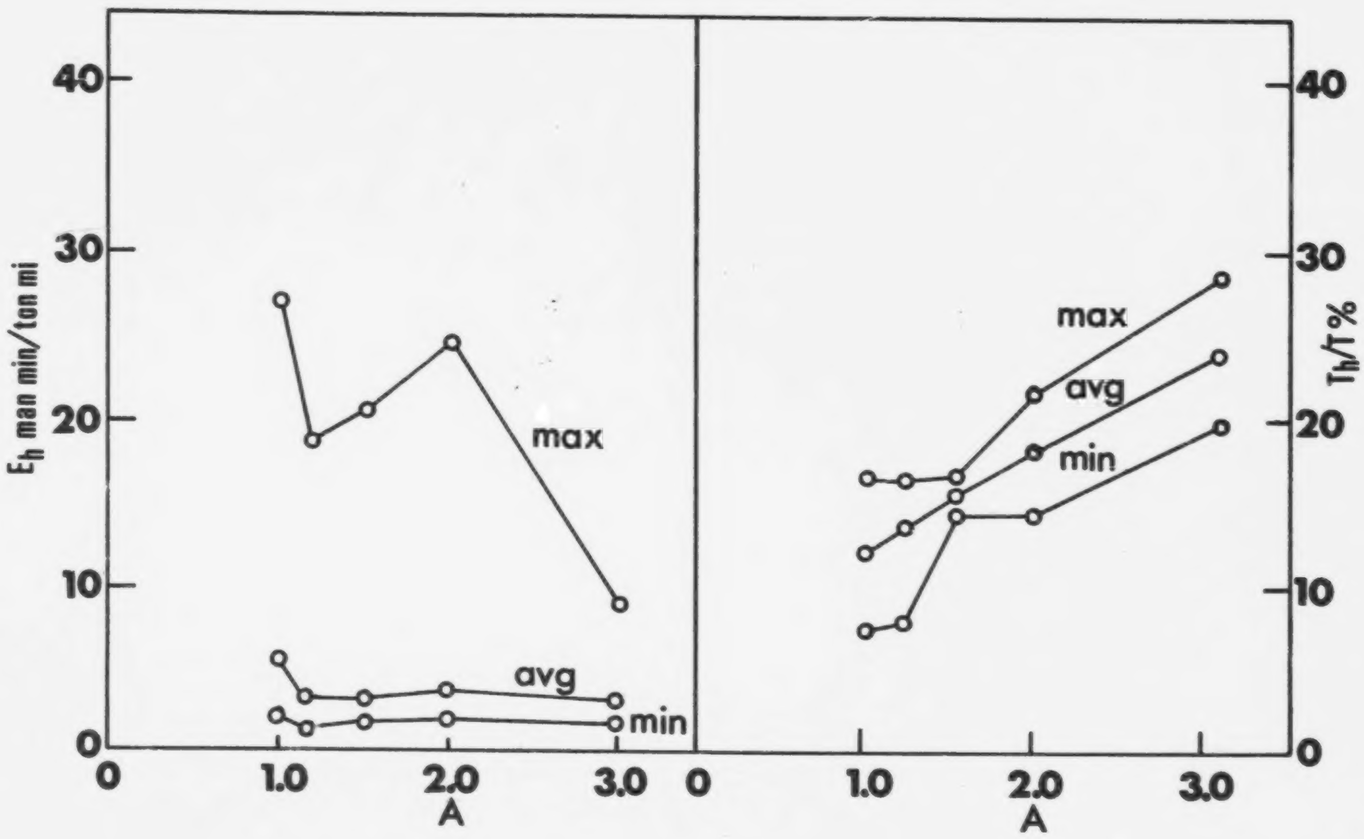

Figure 2-12 Haul Efficiency and Haul Time vs. the Assignment

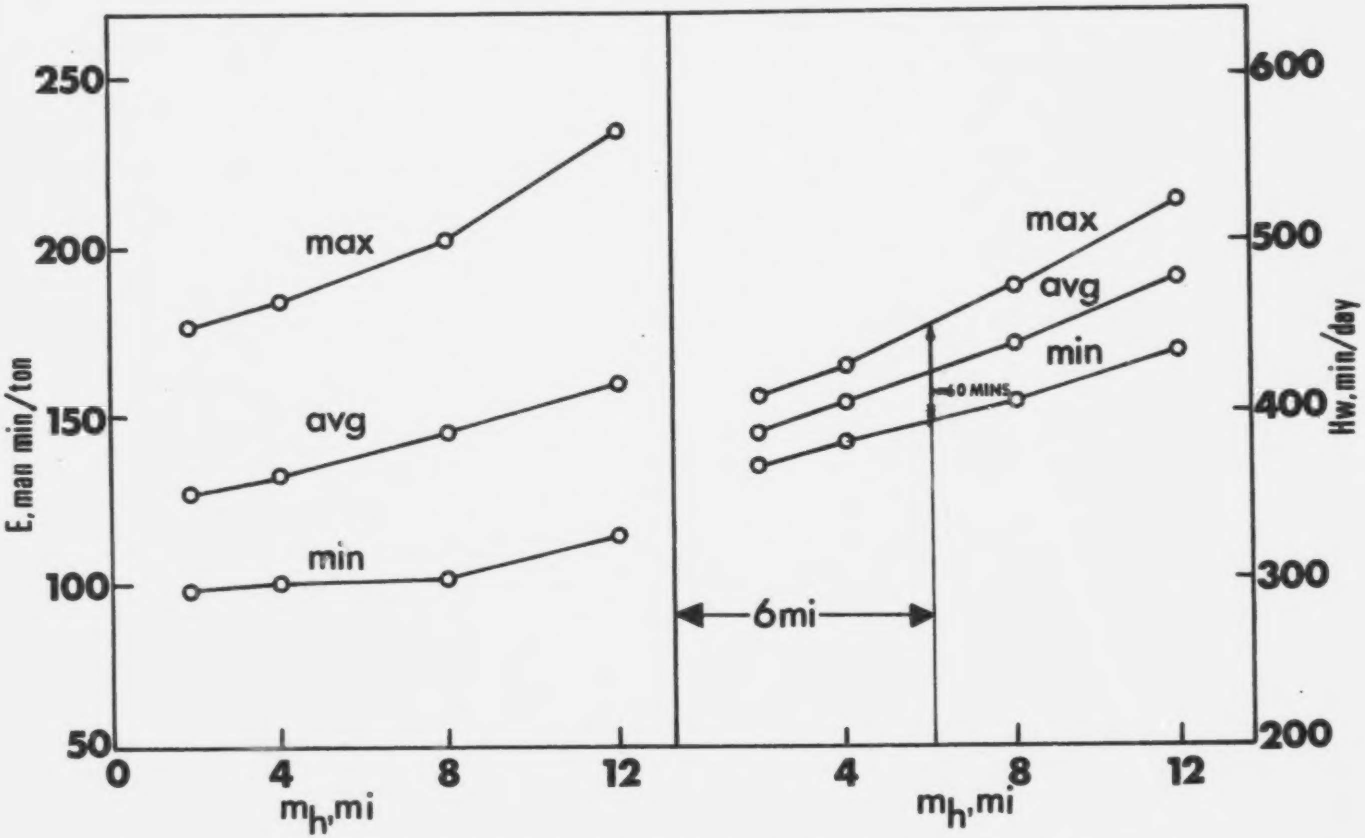

Figure 2-13 Overall Collection Efficiency and Length of Workday vs.One-way ℓ_1-metric Haul Distance

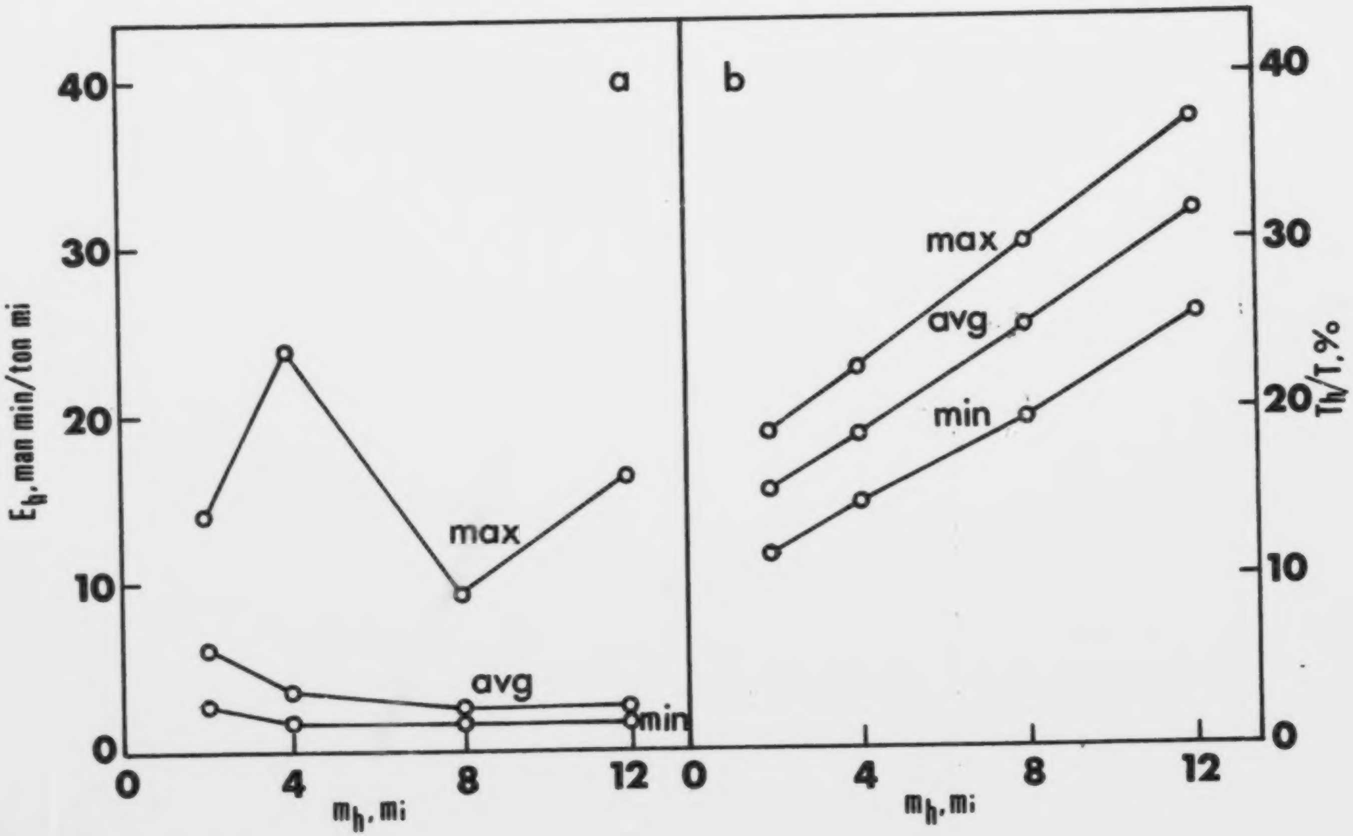

Figure 2-14 Haul Efficiency and Haul Time vs. One-way ℓ_1-metric Haul Distance

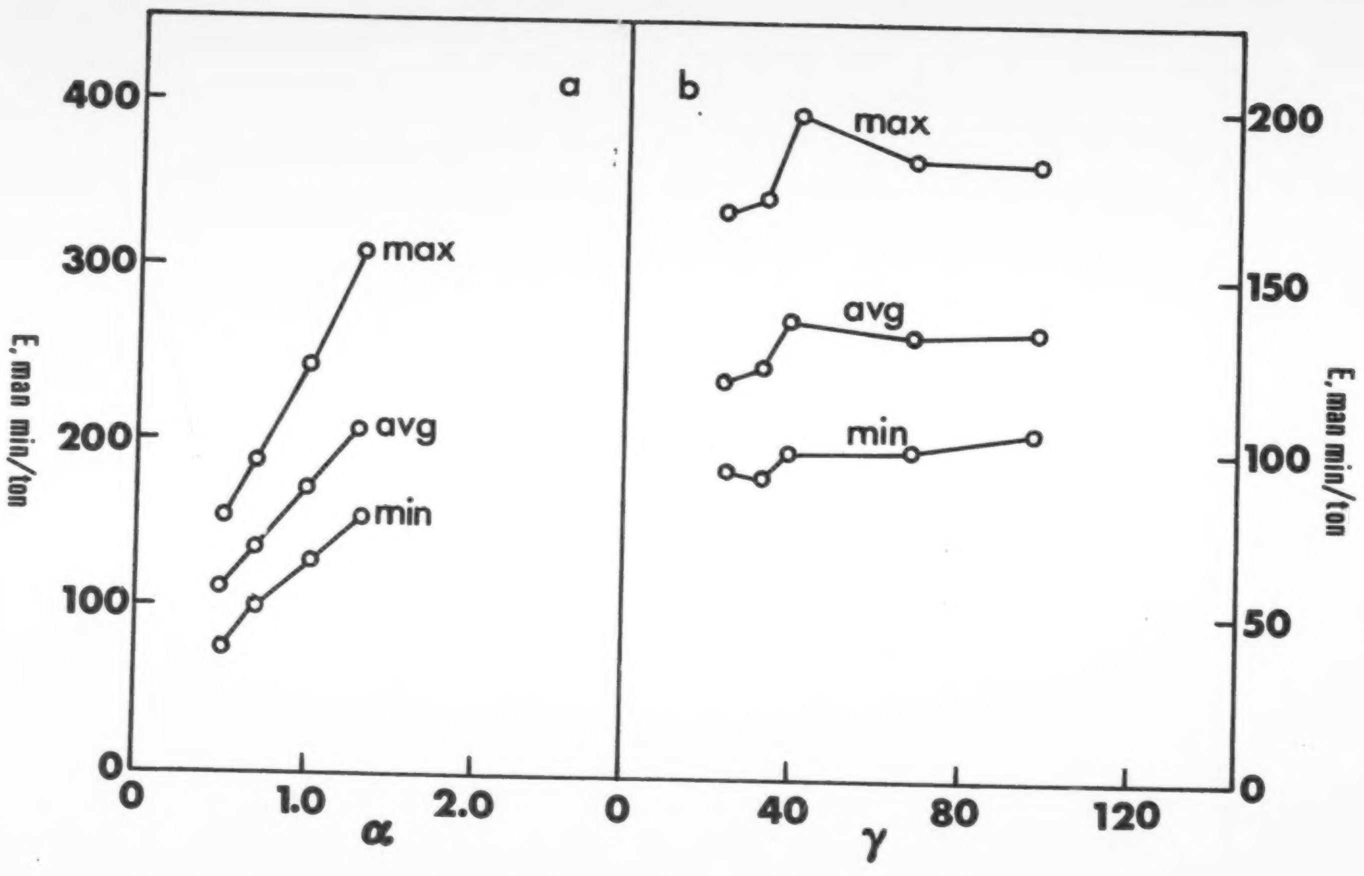

Figure 2-15 Correlation of Overall Collection Efficiency with the Pickup Coefficient

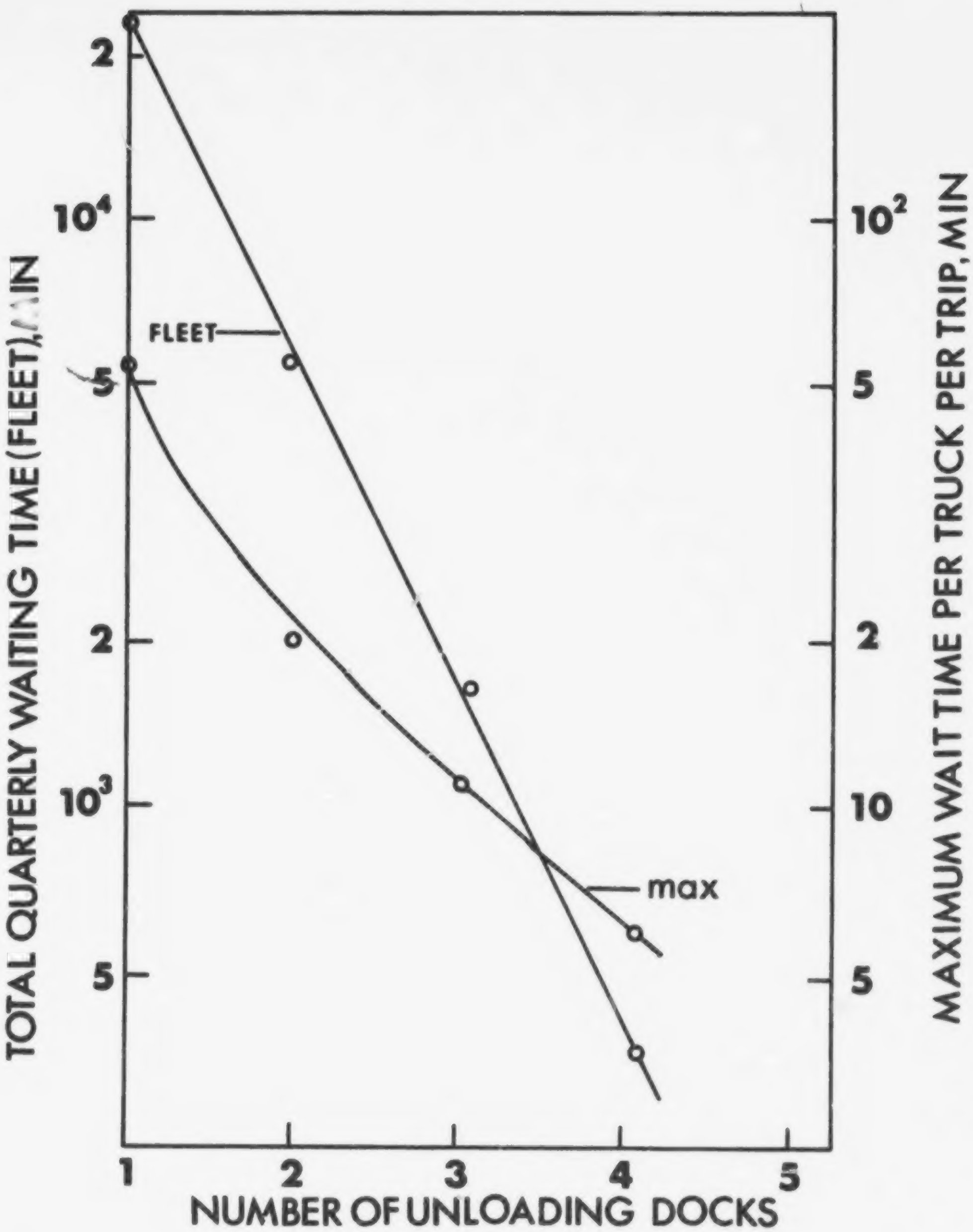

Figure 2-16 Queuing Problem **at** the Disposal Site: Waiting Time per Day vs. the Number of Unloading Docks

The other derived parameters appearing in the graphs are:

1) E_c, truck capacity utilization expressed as percent. This is related to the parameters of Table 2-3 by the formula

$$E_{c\cdot} = \ell\cdot/C \quad 100\% \qquad (4.2)$$

where "•" stands for minimum, maximum or average with the same choice to appear for each "•" in any use of the formula.

2) $T = H_{w\cdot}$, used as the denominator in computing haul time as a percentage of total time worked per day.

$$H_{w\cdot} = \alpha\, S + \gamma + T_{h\cdot} + N\cdot \times t_{d,avg} = T \qquad (4.3)$$

where α, γ have been explained in Chapter 3 and all **other quantities** appear in Table 2-3 with the "dot" convention the same as immediately above.

The simulated behavior of six performance parameters as a function of the coefficient of variability of refuse production (daily fluctuation) for the reference system described in sections A and B is shown in Figures

2-7 through 2-9. The value of σ/μ for Winnetka, a largely commuter community, ranges from 0.067 to 0.265 as shown in Table 2-1. Thus, the lower halves of these graphs represent to some extent the behavior of these parameters in Winnetka while the upper halves represent investigation of the suitability of the daily route method for higher values of σ/μ. The only previous investigation of the influence of σ/μ appears in [13] of which this dissertation is an extension.

Figure 2-7a shows that truck capacity utilization is not greatly influenced by changes in σ/μ. Truck capacity utilization is defined as the ratio of average refuse carried per trip (ℓ) to the truck's capacity both in tons (formula (4.2)). In performing this sort of analysis on a fleet of trucks where capacity is expressed in cubic yards, the compaction ratios of the different trucks must be taken into account. The minimum truck capacity utilization curve ($E_{c,min}$) was interpreted in [13] to mean "that small loads are unavoidable to a large extent and cannot be reduced .. by a change in the boundaries of the collection units to effect an increase or decrease in σ/μ. Small loads can be avoided only for $\sigma/\mu < 0.1$." With the added evidence obtained from the consideration of the frequency histograms of capacity utilization in Tables 2-4a and 2-4b, we must amend this conclusion. Considering constant values of the assignment A, we see that small loads are present at all levels of σ/μ with a pronounced tendency to become bimodal (two peaked) as either σ/μ or A decreases. Or expressed differently, as either σ/μ or A increases an initially bimodal distribution with a wide separation of peaks tends to have the lower peak move toward the higher capacity utilization peak with the distribution between the peaks gaining more frequency and flattening out. We conclude, then, that σ/μ has nothing to do with small loads. Small loads are, rather a function of the capacity rule and the strictures of the daily route method. This is confirmed by Table 2-11 appearing in the next chapter showing the distribution of the number of trips at given capacity utilizations under constant length workday rules with a different capacity rule. Although there are a few small loads, the smallest of these is much larger than the smallest loads under the present conditions.

Figure 2-7b shows that the average daily number of trips N_{avg} is essentially constant as σ/μ varies. The vertical lines through this curve show the range of the minimum and maximum values. This one figure emphasizes that working with average values sheds no light on the lows and highs of effort required to provide a given level of service to a community.

Figure 2-8a reveals that the over-all efficiency, E, in man-minutes per ton is affected by σ/μ . A wide variation in E is seen at large values of σ/μ and this suggests that as in Winnetka, the daily route method is not suited to situations with σ/μ in excess of 0.3. As an example, cutting the curves of Figure 2-8a at $\sigma/\mu = 0.15$ shows a range of E from 95 to 205 man-minutes per ton. The implications of this range can be gauged from Figure 2-8b showing the variation in length of workday with σ/μ . If operational policy requires that the length of the workday not vary by more than 30 minutes above or below the average then one is advised not to use the daily route method unless $\sigma/\mu \leq 0.15$ as indicated on the figure. Further light is shed on the subject of length of workday by considering Table 2-5 showing the average frequency histogram (run and repeat run) of length of workday for all twenty-five cases. It is clear that the effects of σ/μ on the spread of the histogram are most pronounced when the assignment A is high. This suggests that high assignments do not go well together with high variability under the daily route

$P \leq H_w < Q$ (Hrs)		Case (Includes Repeat Run)																								
P	Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	4^5												52													
4^5	5^0																									
5^0	5^5																							25^5		
5^5	6^0			1^5	26^5	30	26^5			3^5	24^5	26							3^5	9^5	16	12^5	21^5	25^5		
6^0	6^5		1	6^5	24	18^5	22^5	24^5	20	21^5	17	26			26			48	41	31	34^5	36^5	26	1		
6^5	7^0	10	6	6^5	1^5	3^5	3	8^5	13^5	7^5	10^5				26	23		4	5^5	7	1^5	3	3			
7^0	7^5	18	20	11				19	17^5	15^5						2^5	22		2	4^5						
7^5	8^0	23^5	18	17^5					1	4						26^5	4									
8^0	8^5	5	7	7^5													6^5								25^5	
8^5	9^0			1													19^5								26^5	
9^0	9^5				5									26												
9^5	10^0													25^5												24
10^0	10^5														5											11
10^5	11^0																									17
	S							350					200	500												
	ρ_s							35			70	100														
	m_h							4							2	8	12									
	μ	15	15	15	5	5	5	10																		
	σ/μ		0^3	0^6		0^3	0^6	0^1	0^3	0^6									0^3	0^6	0	0^3	0^6			
	α							0^7																0^5	1^0	1^3
	C							3										4	4	4	5	5	5			
	A	3^{06}	3^{06}	3^{06}	1^{02}	1^{02}	1^{02}	2^{04}					1^{17}	2^{92}				1^{53}	1^{53}	1^{53}	1^{23}	1^{23}	1^{23}			

Blank Indicates
Standard Value of
Cases 7, 7R

Table 2-5 Frequency Distribution of Lengths of Workday and Settings of Input Values of 25 Cases

method if controls of this sort are desired.

The variation of haul efficiency, E_h , measured in man-minutes per ton-mile is shown in Figure 2-9a. It is clear that the average and minimum values of E_h are unaffected by variations in σ/μ . The maximum curve of E_h reflects via its appearance in the denominator the effects of small loads. Contrary to the conclusion reached by [13] the apparent hump at $\sigma/\mu = 0.3$ is not some special property of $\sigma/\mu = 0.3$ but is simply caused by the random appearance of a disproportionate number of elevated $E_{h,max}$ figures for the cases having $\sigma/\mu = 0.3$. If, instead of computing

$$E_{h,max} = \frac{T_{h,max}}{2m_h \cdot L_{max}} \qquad (4.4)$$

we take the readings of E_h directly from the computer printouts and eliminate those $E_{h,max}$ which are clearly out of line and take a weighted average where the weight is the number of trips for the given run, we get a curve of the appearance of max in Figure 2-9a. (This has not been done for the minimum and average curves.) It should also be pointed out that there is no evidence in the partial load histograms of Tables 2-4a and 2-4b to support a special status for $\sigma/\mu = 0.3$.

Figure 2 9b depicts the relationship between haul time as a percentage of total time (daily) and σ/μ. One expects and finds these curves to be similar to those of Figure 2-7b, i.e., they closely reflect the behavior of the daily number of trips under changes in σ/μ. The analysis of this data in [13] stated that the nondependency of the average curve on σ/μ "suggests that the number of partial loads is the same for all $\sigma/\mu > 0.1$." The evidence available from Table 2.4b suggests that while this is not precisely so, there are enough partial loads at both $\sigma/\mu = 0.3$ and 0.6 to account for this effect. At the highest level of assignment $A = 3.06$, the cumulative distributions for both $\sigma/\mu = 0.3$ and $\sigma/\mu = 0.6$ are pretty close (Table 2-6).

We turn next to analysis of the effects of the assignment A which has been defined above. Figure 2-10a shows the relationship between E_c and A. This graph taken together with Tables 2-4a and 2-4b indicates that increasing the assignment up to $A = 2$ (particularly at low levels of σ/μ) increases capacity utilization by shifting upward and smoothing out the initially bimodal density. The underlying mechanism causing this shift is not apparant from the simulation results. Above $A = 2$, the distribution of E_c continues to flatten but not as

$E_c < Q$	CUMULATIVE DISTRIBUTION	
$Q\downarrow$	$\sigma/\mu = 0.3$	$\sigma/\mu = 0.6$
12%	1.1	0.3
24%	4.5	4.4
36%	7.1	7.4
48%	10.8	10.1
60%	14.2	13.4
72%	17.4	18.0
84%	22.2	23.2
96%	69.2	65.3
108%	100.0	100.0

Table 2-6 Cumulative Distribution of E_c for $\sigma/\mu = 0.3, 0.6$ with $A = 3.06$.

dramatically. Thus, A is not the cause of small loads. Again, we must point at the daily route method itself and our capacity rule.

Figure 2-10b shows the increase in the number of trips necessary to service the assigned area as the quantity of refuse expressed in truckloads increases. The linear behavior of the average curve is, of course, expected. The variations above and below average due to variable refuse production are also expected. What is not expected is the lack of variation at $A = 1.5$. Changes in A are due to changes in the number of services, changes in the refuse production rate or changes in the size of the truck. In the present instance, all the cases at $A = 1.5$ coincide with truck capacity $C = 4$. It is possible that this capacity bears some felicitous relation to the other parameters of the base case that enable it to "swallow" the effects of variation. There is nothing about Table 2-4a , b that would seem to shed light on this and the matter warrants further investigation.

The relationship between E and A shown in Figure 2-11a is predictable on theoretical grounds. Using a daily basis and letting the crew size be denoted by K we may write a "rational" formula for E as

$$E = \frac{(\alpha S + \gamma)}{CA} + \frac{2m_h K}{CV} + \frac{t_d K}{C}$$

whence

$$[\ E - \frac{K}{C}(\frac{2m_h}{V} + t_d)]\ A = \frac{1}{C}(\alpha S + \gamma) \qquad (4.5)$$

which is a rectangular hyperbola with asymptotes $E = K(2m_h/V + t_d)/C$ and $A = 0$. Thus, we note that beginning at $A = 1.2$ some other mechanism of the system causes the curves to be depressed below the curve predicted by (4.5). As A increases from 2.0 to 3.0 the effect of this mechanism abates and the approach of the curve to the asymptote is reestablished. For the average curve, (4.5) becomes $(E - 25)\ A = 207$ so that for $A = 1.0, 1.2, 1.5, 2.0, 3.0$ we should have $E = 232$ (close), 198 (low), 163 (low), 128 (close), 94 (close). Again, the underlying mechanism of this anomaly seen in all three curves is unclear and warrants further investigation. It is likely that this behavior is related to the behavior of $A = 1.5$ noted above in Figure 2-10b and is related to the constriction of the minimum, average, and maximum curves of H_w as a function of A seen in Figure

2-11b. The cause put aside, one may still conclude that for the reference system an assignment of $A = 1.5$ provides the best system performance in terms of constancy of the number of trips per day, length of workday and lowest over-all collection efficiency. This conclusion is confirmed by the results shown in Figures 2-12a and b. The former (with the maximum curve not corrected for small loads) shows that $A \geq 1.2$ provides lowest and nearly constant haul efficiency; the latter shows that for $A = 1.5$, the minimum, average, and maximum values of the proportion of haul time are nearly constant.

We turn next to four graphs which exhibit the behavior of E, H_w, E_h, and T_h/T with changes in m_h, the one-way haul distance from the collection unit to the disposal site. It is to be recalled that distances in both simulation models are computed by the ℓ_1-metric (L-shaped paths). The changes in m_h are made while holding all other parameters at the base values as shown in Table 2-2 . Figures 2-13a and b show that both E, the over-all collection efficiency, and H_w, daily time worked, increase with the haul distance. The smooth linear appearance of the average curves conforms to the linear dependence of the underlying rational formulas on m_h . The divergence of the minimum and maximum curves can only be ascribed to random elements introduced into the haul speed as all points in these figures correspond to $\sigma/\mu = 0.1$ and $A = 2.0$. We note, also, from Figure 2-13b that for the reference system, the requirement that the length of workday not vary by more than 30 minutes above or below the average indicates that the one-way haul distance should not exceed 6 miles. Some additional light is shed on the effect of m_h on H_w by returning to Table 2-5 and considering cases 14, 7, 15, and 16 in that order. We note that not only does the workday length increase with m_h , but that the distribution of the length of workday spreads out and becomes pronouncedly **bimodal** with almost all days appearing equally divided between the two end intervals. We suggest that the reason for this is that the actual inequitable division of workload between the two collection days of the schedule (one getting 3 day's worth and the other 4 day's worth of refuse) is masked by the smaller haul distances. The lower hump represents the 3 day collection, the upper hump represents the 4 day collection and the separation between the humps represents the increasing proportion of the workday devoted to the haul operation. This latter effect is confirmed by Figure 2-14b.

Figure 2-14a (with the maximum curve not corrected for small loads) shows in the average and minimum curves that haul efficiency tends to level out for $m_h > 4$ miles since the increase in haul speed with distance compensates for the longer haul distance. The curve of speed vs. distance is discussed in Appendix 2-A. There it will be seen that $m_h = 2$ is handled differently than the other values of m_h appearing in Figures 2-13 and 2-14 . The maximum curve shows again, the effect of small loads. There is nothing in the capacity utilization histogram data to indicate any alteration of distribution under changes in m_h .

Figures 2-15a and b reflect the dependence of T_p on α and γ via the equation (3.1) of Part 2 $T_p = \alpha S + \gamma$. These graphs indicate that α (service factor) has a stronger influence on E than does γ (density factor). This is merely a function of the way the model was constructed to emulate a situation (Winnetka) in which refuse amount had nothing to do with pickup time, but the number of services was dominant. The three curves in Figure 2-15a meet at the E-axis at the common value $\gamma = 33$. The anomalous behavior of the three curves of Figure 2-15b at $\gamma = 40$ warrants further investigation. Whatever function is relevant to describe T_p as a function of observable factors, results such as those of Figures 2-15a and 2-15b allow estimates to be made of the value of various innovations in technology and/or policy via their predictable effects on the coefficients α and γ or whatever expanded set of coefficients is relevant.

Figure 2-16 shows total quarterly waiting time for a fleet of 13 trucks (described in section B) as a function of the number of unloading docks For the system being simulated, a ratio of one unloading dock to every 3 or 4 trucks seems to bring the waiting time down to an acceptable level. This is reflected, likewise, by the plot of the maximum waiting time per truck per trip. This curve shows that the maximum wait is reduced from 10 minutes with 3 unloading docks to about 7 minutes by the addition of a fourth dock. Considering the fleet total, the fourth unloading dock saves 237 man hours per year.

APPENDIX TO CHAPTER 4

NOTATION

A = the assignment, quantity of refuse assigned per day expressed in number of full truckloads per day;

C = truck capacity, tons;

C_e = average effective capacity of the truck; tons;

CU = collection unit;

E = overall collection efficiency, man-min per ton;

E_c = average capacity of the truck utilized, %;

E_h = average haul efficiency, man-min per ton-mi;

E_p = pickup efficiency, man-min per ton ;

F = frequency of collection per service per week ;

H_w = length of workday, min or hrs ;

K = crew size, number of men per truck ;

L = quantity of refuse collected per day, tons ;

L_a = the load, quantity of refuse collected per day expressed in number of full truckloads per day ;

ℓ = quantity of refuse collected per trip, tons;
M_i = internal mileage travelled per day, mi;
M_h = haul mileage per day, mi;
m_i = internal mileage travelled per trip, mi;
m_{gd} = mileage between garage and disposal site, mi;
m_h = one way distance (ℓ_1-metric) from a collection unit to the disposal site, mi;
N = number of trips per day;
ΣN = total number of trips in a quarter;
S = number of services;
T_h = haul time per day, min;
T_p = pickup time per day, min;
t_d = time at the disposal site, per trip, min;
t_{dw} = waiting time at the disposal site per trip, min;
t_p = pickup time per trip, min;
ΣT_d = total time at the disposal site in a quarter, min;
ΣT_h = total haul time in a quarter, min;
ΣT_p = total pickup time in a quarter, min;
V = average speed of the truck during haul, mph;
α = a constant characteristic of the time needed to collect refuse from a service, min/service;

γ = a constant characteristic of the time spent in traveling within the collection unit, min;

μ = refuse production, lb/service/day;

ρ_s = service density, number of services per mile;

σ = standard deviation of the refuse production;

θ = pickup time for S services, min.

CHAPTER 5

Simulation Studies of Alternative Policies in a Constant Length Workday System [22]

In this chapter we present studies made on a constant length workday refuse collection system based on the City of Chicago. The daily workflow and the chief differences between this model and the model used in the previous chapter has been discussed in Section B of Chapter 3.

The aim of these studies is to examine system performance in terms of its quality under varying policies. These policies are last load relay as defined above (yes or no), allowance of overtime (yes or no, but see below for clarification), and assignment (six levels). We are concerned here with two kinds of assignment. The first is the refuse assignment A_r which was used under the symbol A in the previous chapter. This is the average daily amount of refuse expressed as number of truckloads. The second kind of assignment is the time assignment A_t which is the average daily hours assigned expressed as number of 8-hour periods. (A glossary of notation used in this chapter appears at the end of the chapter as an appendix.)

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[22] See [15].

In the previous chapter we were performing parametric studies so that a great many input parameters were varied. Here almost all variables that can be varied are held to a single input value and the changes range instead over the policy choices.

Section A

The Reference System under Constant Length Workday

The invariant parameters of the system simulated are

1) A constant length workday.

2) Collection crew consisting of three loaders and a driver.

3) Collection units are serviced once a week. The work week is Monday through Friday. Each truck services two CU's per day (ten per week) and each crew is limited to a maximum of two trips per day. Work may be left over to tomorrow or beyond.

4) The number of services per CU is 180 and the service density, ρ_s , is 90 services mile.

5) The average quantity of refuse produced, μ , is 8 pounds per service per day with a standard deviation $\sigma = 6.4$ pounds per service per day. The normal density is truncated at the left for a minimum refuse production of 4 pounds per service per day. On a weekly

basis, the mean, standard deviation and minimum are respectively 56(= 8 x 7), 17(= 6.4 × $\sqrt{7}$) and 28(= 4 × 7). These values are characteristic of high population density, low per capita income areas [6-D2].

6) The time spent at the disposal site is uniformly distributed between 5 and 15 minutes or 2 and 6 minutes. The queuing time at the disposal site is handled as in the previous model.

Pickup time computation and the decision criteria were discussed in section B of Chapter 3 of Part II, and the rest of the inputs are summarized in Table 2-7. The geographical layout of the CU's and the disposal site is depicted in Figure 2-17. Each day begins and ends at the garage and travel speeds are computed as shown in Appendix 2-A.

Input Parameter	Value
Total number of services	1800
Service density, ρ_s, in services per mile	90
Haul distance, m_h, in miles	(see Fig. 2.17)
Average quantity of refuse, μ, in pounds per service per day	8
Standard deviation for quantity of refuse, σ, in pounds per service per day	6.4
Pickup time coefficient, α, in min per service	0.7
Pickup time coefficient, β, dimensionless	1.0
Pickup time coefficient, γ, in min	33
Pickup time distribution parameter, $(b - a)$, in min	0.4
Maximum truck capacity, C, in tons	5.5
Decision criteria at collection unit	
CLT 1, in min	15
HIRI, as fraction of C	0.9
Decision criterion at disposal site, CLT 2	
constant, δ, in min	20
coefficient, ϵ, fraction	0.2

Table 2-7 Summary of Values of Input Parameters

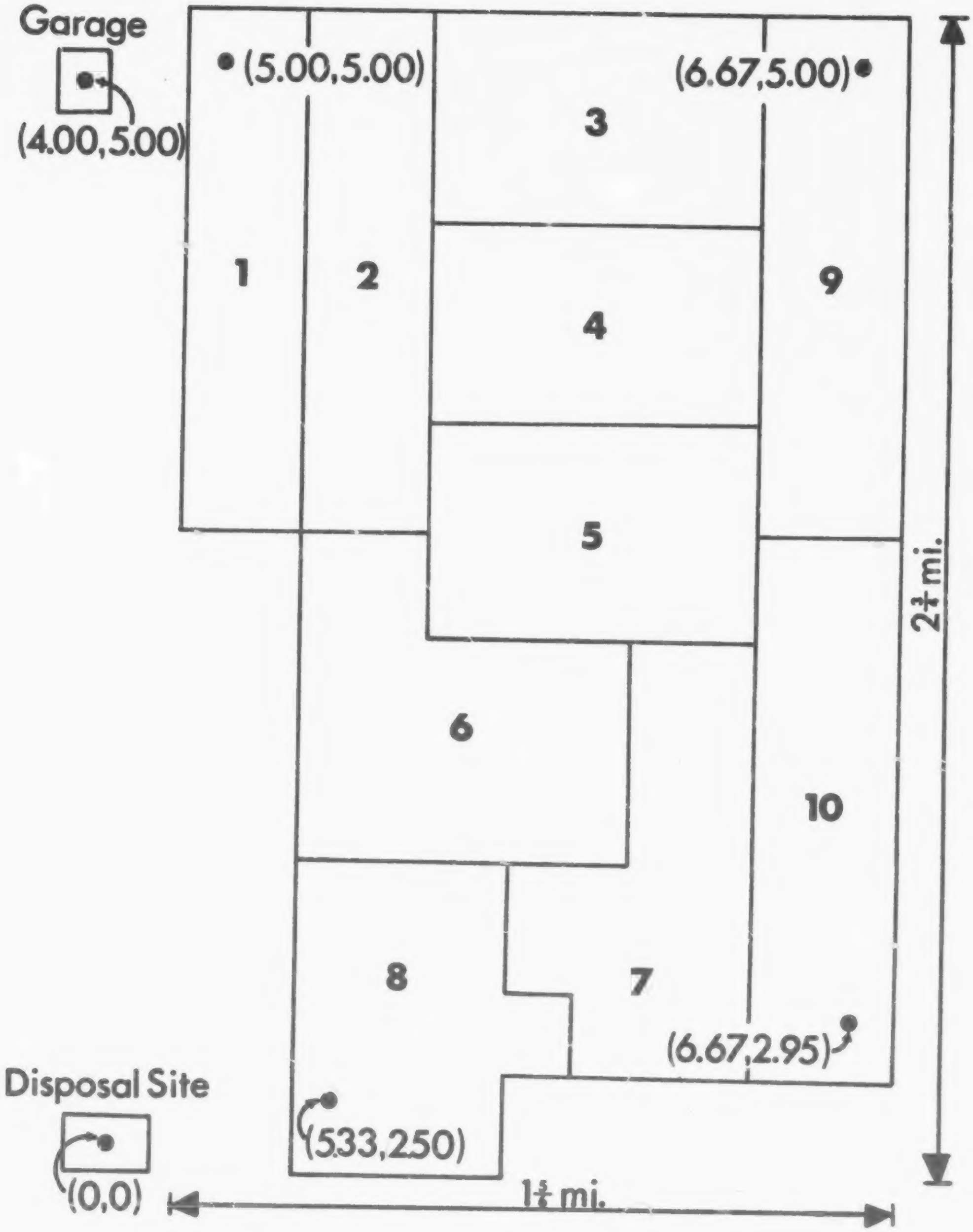
Garage
(4.00,5.00)
(5.00,5.00)
(6.67,5.00)
1
2
3
4
5
6
7
8
9
10
(6.67,2.95)
Disposal Site
(0,0)
(5.33,2.50)
1½ mi.
2½ mi.

Figure 2-17 Hypothetical Collection Area

Description of Alternate Policies

Table 2-8 summarizes the alternate policies of relay, overtime and assignment under investigation and introduces the symbols for them to be used in the subsequent tables and discussion.

The last load relay policy as noted previously is governed by the setting of the decision criterion CLT 3. under this policy, the last load of the day is delivered to the garage whence the relay driver takes it to the disposal site allowing the three loaders and regular driver to quit within the allotted workday (assignment A_t). Relay policy can be and is hereinunder combined with the overtime policies. The overtime policy is specified by three overtime parameters OT1, OT2 and OT3 denoting respectively number or hours overtime permitted to service yesterday's (or earlier), today's, and tommorrow's refuse assignment. The two policies used here are P_1 : no overtime in any category; and P_2 : two hours allowed for yesterday's and today's collection, but no overtime allowed for living units (LU's) scheduled for tomorrow.

Policy	Information Associated with Policy		
Relay	Relay		
R_1	no		
R_2	yes		
Overtime	OT1	OT2	OT3
P_1	0	0	0
P_2	2	2	0
Assignment	F	A_r	A_t
A_1	7	2.28	1.16
A_2	7	2.28	1.10
A_3	8	2.00	1.05
A_4	8	2.00	0.98
A_5	9	1.78	0.96
A_6	9	1.78	0.90

Table 2-8 Summary of Operational Policies Considered

The collection and haul costs relevant to these policies were based on hourly wage rates of \$4.00 for the regular driver,\$3.75 for each of the three loaders, \$4.00 for the relay driver, and on a daily truck rental of \$32.00. An eight hour day is charged for regular members of the crew with overtime billed at time-and-a-half for the actual time involved. The relay driver is billed on a straight time basis.

The assignment policies are chosen to center about a refuse assignment, A_r , of 2 and about a time assignment, A_t , of 1. These are respectively the expected refuse amount in truckloads and the expected workday length in units of eight hours. In the cases run, the assignment was varied by changing the fleet size, F , from 7 through 9 trucks. As will be seen, both the refuse and the time assignments affect the quality of service and the need for overtime.

Section C

Results and Analysis

The types of outputs provided by the simulation model shown in Table 2-9 are quite similar to those provided by the first model (Table 2-3). What has been added is ΣT_o , the time in minutes spent in overtime work over the quarter; Σn_o , the number of days during the quarter (out of a fixed total of 65 days) when overtime work was required; and an entirely new section entitled "Quality of Service Provided." This section reports the percentage of collection units serviced on time (Q_1), the percentage of collection units serviced one day late (Q_2), and the percentage of collection units serviced one day early (Q_3). The latter category generally appears only at zero level in the results reported here since OT3 is kept at zero in both overtime policies investigated. The percentage of collection units not serviced (i.e., serviced two or more days late), Q_o, can be obtained by subtracting $Q_1 + Q_2 + Q_3$ from 100%.

Further insight is obtained from frequency histograms of the length of workday and capacity utilization. Examples are shown in Tables 2-10 and 2-11 . This is the same type of information which was given in the previous chapter in Tables 2-4a, 2-4b and 2-5.

	Value or Average	Minimum	Maximum	Standard Deviation
Time Distribution				
total pickup time, ΣT_p, in %	82.0			
total haul time, ΣT_h, in %	15.1			
total time at disposal site, ΣT_d, in %	2.9			
pickup time per trip, t_p, in min	191	93	312	42
haul time per day, T_h, in min	70	71	83	5
time at disposal site per trip, t_d, in min	14	6	52	7.3
total overtime, ΣT_o, in min	26			
Crew Performance				
hours worked per day, H_w, in hours	7.77			
pickup efficiency, E_p, man-min per ton	153			
over-all collection efficiency, E, man-min per ton	188			
refuse collected per day, L, in tons	9.95	6.94	11.0	1.14
number of days worked overtime, Σn_o				
Truck Utilization				
total trips in quarter, ΣN	130			
trips per day, N	2			
haul speed, $60V$, mph	19.0			
effective truck capacity, C_e, in tons	5.00			
refuse collected per trip, ℓ, in tons	4.98	1.86	5.50	0.94
internal distance traveled per trip, m_i	3.98	3.31	4.24	0.24
Quality of Service Provided				
% of collection units serviced on time, Q_1	76.2			
% of collection units serviced one day late, Q_2	23.8			
% of collection units serviced one day early, Q_3	0.0			

Table 2-9 Example Output Information from Simulation Model[a]

[a] Time Span: 13 weeks; Operating Policies R_2, P_1, A_4; truck no. 1

Length of Workday, Upper Limit, hrs.	Number of Days In Interval.
4.5	0
5.0	0
5.5	0
6.0	0
6.5	0
7.0	3
7.5	11
8.0	29
8.5	22
9.0	0
9.5	0
10.0	0
10.5	0
11.0	0
11.5	0

Table 2-10 Example Output Histogram of Length of Workday[a]

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[a] Time Span: 13 weeks; Operating Policies R_2, P_1, A_4; truck no. 1

Capacity Utilized Upper Limit	Number of Trips In Interval	Capacity Utilized Upper Limit	Number of Trips In Interval
4	0	64	0
8	0	68	10
12	0	72	1
16	0	76	1
20	0	80	3
24	0	84	5
28	0	88	0
32	0	92	4
36	2	96	4
40	0	100	91
44	1	104	0
48	1	108	0
52	3	112	0
56	4	116	0
60	0	∞	0

Table 2-11 Example Output Histogram of Truck Capacity Utilization[a]

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[a] Time Span: 13 weeks; Operating Policies R_2, P_1, A_4; Truck No. 1

Costs of the service provided are computed from the wages and rentals cited in section B on three bases: the total cost of collection, C_t, in dollars per week; the cost in dollars per living unit (service)per week ($/LU/wk), C_1; and cost in dollars per ton of refuse collected C_2.

Table 2-12 shows the effect of relay policy when no overtime is used and the assignment policies are A_3 and A_4[23/]. Without allowance for overtime, the effect of relay as compared to no relay is to increase the total cost, C_t, and the unit cost, C_1. This increase is due to the added cost of the relay driver. The unit cost C_2 is seen to decline, but this may be ascribed to random fluctuation in the amount of refuse produced. There is no such random fluctuation in the number of living units so that the unit cost, C_1, is proposed as a more reliable index of the cost effectiveness of policy changes.

At first sight, the difference of .11 $/LU/wk between relay and non-relay seems insignificant, but this can amount to .57 $/LU/yr or 1026 $/truck/yr based on the 1800 living units served by a truck in this example. There is, of course, a great deal of difference between the quality of service provided and a citizen might well

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[23/] The relay effect dominates the assignment effect here.

Operating Policies P_1,

Item	Relay Policy	
	A_3, R_1	A_4, R_2
Cost of Service		
Total cost, C_t, \$/wk	6,150	6,300
Unit cost, C_1, \$/LU/wk	0.427	0.438
Unit cost, C_2, \$/ton	16.7	15.9
Quality of Service[a]		
Percentage of collection units serviced on schedule, Q_1	6.2	76.2
Percentage of collection units serviced one day late, Q_2	83.8	23.8
Percentage of collection units serviced one day early, Q_3	0.0	0.0
Percentage of collection units not serviced, Q_0	10.0	0.0

[a] Time Span: 13 weeks

Table 2-12 Effect of Relay Policy on the Quality and Cost of Service Provided

consider 57 cents a year a bargain for having his refuse picked up on time three times out of four rather than (roughly) one time in twenty (and very late one time in ten).

The major factors involved here are the amount of time available for collection (increased by relay), the average refuse production (μ) and the variability of refuse production (σ). The no-relay policy, R_1, yields poor performance because the 8-hour time assignment leaves insufficient slack to absorb the shocks of variations of refuse production above the average. (Recall that we are using a large ratio $\sigma/\mu = 0.8$ and the effect of truncation at the left is to skew the average to the right.) There are of course other variations in performance (pickup rate, wait at the disposal site, travel time, etc.) for which the provision of slack time would be useful. It is apparent that the extra time provided by relay under policy R_2 is a good dose of slack time. Another factor relevant here is the capacity of two loads per day of 11 tons is not sufficient to accommodate the variations in refuse production without the use of relay.

The effect of overtime on the cost and quality of services is shown in Table 2-13. The availability of

Operating Policies R_2, A_4

Item	Overtime Policy	
	P_1	P_2
Cost of Service		
Total cost, C_t, \$/wk	6,300	6,340
Unit cost, C_1, \$/LU/wk	0.438	0.438
Unit cost, C_2, \$/ton	15.9	16.1
Quality of Service[a]		
Percentage of collection units serviced on schedule, Q_1	76.2	58.5
Percentage of collection units serviced one day late, Q_2	23.8	0.0
Percentage of collection units serviced one day early, Q_3	0.0	41.5
Percentage of collection units not serviced, Q_0	0.0	0.0

[a] Time Span: 13 weeks

Table 2-13 Effect of Overtime Policy on the Quality and Cost of Service Provided

slack provided by overtime is not needed since the no-overtime system already has the slack provided by relay. All the relay plus overtime system can find to do with the superfluous time is service many collection units a day early. Very little overtime is actually used and the unit cost comes out nearly the same. A better comparison on overtime is shown by cases 5 and 6 of Table 2-15 . There, an improvement in quality is purchased at a large increase in the unit cost (C_1) .

The cost effectiveness of improvement in the quality of service is shown by Table 2-14 . Here we see that going from assignment A_2 to A_4 we get an 81% improvement in on-time service for an increase of total cost per week of $800. Going from assignment A_4 to A_6 we pay an additional $800 per week but only get an additional 12% improvement in on-time service: a prime example of the "law of diminishing returns".

Twelve of the twenty-four possible combinations of policy were run and the results arranged by decreasing assignment are shown in Table 2-15 . The table includes unit costs, quality of service (plus an over-all rating), and report of time worked. The values of H_w are average daily hours worked over the 65 days of a 13 week period; ΣT_o and Σn_o are for the same period the total

Operating Policies R_2, P_1

Item	Assignment Policy		
	A_2	A_4	A_6
Cost of Service			
Total cost, C_t, \$/wk	5,510	6,300	7,090
Unit cost, C_1, \$/LU/wk	0.386	0.438	0.494
Unit cost, C_2, \$/ton	15.8	15.9	16.5
Quality of Service[a]			
Percentage of collection units serviced on schedule, Q_1	5.4	76.2	87.7
Percentage of collection units serviced one day late, Q_2	83.0	23.8	12.3
Percentage of collection units serviced one day early, Q_3	0.0	0.0	0.0
Percentage of collection units not serviced, Q_0	11.6	0.0	0.0

[a] Time Span: 13 weeks

Table 2-14 Effect of Assignment Policy on the Quality and Cost of Service Provided

		Operational	Unit Cost		Quality of Service					Time Worked		
Run		Policies	C_2	C_1	Q_1	Q_2	Q_3	Q_o	Rating	H_w	ΣT_o	Σn_o
1		A_1, R_1, P_1	17.0	0.375	3.1	76.1	0.0	20.8	poor	7.77	0.3	7
2		A_1, R_1, P_2	16.7	0.428	14.6	80.8	0.0	4.6	poor	8.84	61.1	54
3		A_2, R_2, P_1	15.8	0.386	5.4	83.0	0.0	11.6	poor	7.75	0.5	20
4		A_2, R_2, P_2	15.8	0.418	21.7	74.5	0.0	3.8	poor	8.30	34.6	39
5	↑	A_3, R_1, P_1	16.7	0.427	6.2	83.8	0.0	10.0	poor	7.77	0.1	3
6	$A_t > 1$	A_3, R_1, P_2	16.3	0.458	94.6	5.4	0.0	0.0	poor	8.41	32.5	52
7	$A_t < 1$	A_4, R_2, P_1	15.9	0.438	76.2	23.8	0.0	0.0	fair	7.77	0.4	22
8	↓	A_4, R_2, P_2	16.1	0.438	58.5	0.0	41.5	0.0	good	7.83	0.7	30
9		A_5, R_1, P_1	17.4	0.480	79.2	20.8	0.0	0.0	good	7.82	1.7	17
10		A_5, R_1, P_2	16.7	0.480	95.5	0.7	3.8	0.0	good	7.70	1.2	5
11		A_6, R_2, P_1	16.5	0.494	87.7	12.3	0.0	0.0	good	7.53	0.0	6
12		A_6, R_2, P_2	17.2	0.494	93.1	6.9	0.0	0.0	good	7.21	0.1	3

Table 2-15 Cost and Quality of Service for Twelve Operational Policies

hours of overtime and the number of days overtime was required.

One observes that as the assignment decreases going from case 1 to case 12, the quality of service improves and the unit cost, C_1 , increases. The unit cost, C_t , as previously noted is not a reliable index since the quantities of refuse produced are subject to wide variations.

Comparing even- with even- and odd- with odd-numbered runs, we note that except for run 12, decreasing the time assignment causes an improvement in the quality of service. We note also that comparing odd- with even-numbered runs, the use of overtime always improves the quality of service. If we look at the unit costs, however, we note that the improvement gained by use of overtime raises the unit cost, C_1 , when $A_t < 1$ and leaves it the same or nearly so when $A_t > 1$. We conclude that the sagacious use of small amounts of overtime provides improvement in service at slight increase in unit cost when the total time assignment is less than one. It is also apparent from inspection of Table 2-15 that the refuse assignment, A_r , cannot be wholly compensated either by overtime or relay when it is set too large.

The cheapest and next cheapest "good" cases are 8 and 6. Operationally, case 6 may be preferable to the cheaper 8 under a system in which the residents set out their refuse containers on appointed days. Under such a system, early pickup may actually be an inconvenience.

APPENDIX TO CHAPTER 5

NOTATION

The following symbols were adopted for use in this chapter.

A = designation for different assignment policies;

A_r = the average quantity of refuse assigned to a crew per day, in number of truckloads per day;

A_t = the average length of workday assigned to a crew, in eight hour units;

$(b - a)$ = the difference between upper and lower limits of the normalized uniform distribution used in determing pickup time;

C = the maximum truck capacity, in tons;

C_e = average effective capacity of the truck, in tons;

C_t = total cost of collection and haul, in \$/week;

C_1 = unit cost of collection and haul, in \$/LU/week;

C_2 = unit cost of collection and haul, in \$/ton;

CLT 1 = decision criterion for the operation of a truck;

CLT 2 = decision criterion for the operation of a truck;

CLT 3 = decision criterion for the operation of a truck;

CU = symbol for collection unit;

DS = symbol for disposal site;

E = over-all collection efficiency, in man-min per ton;

E_p = pickup efficiency, in man-min per ton;

F = the number of trucks in a fleet;

G = symbol for garage or parking lot;

H_w = average length of workday, in min or hrs;

HIRI = decision criterion for the determination of remaining truck capacity at the completion of each CU;

k = constant, zero to one;

L = quantity of refuse collected per day, in tons;

LU = symbol for living unit;

ℓ = quantity of refuse collected per trip, in tons;

m_i = internal mileage traveled per trip, in miles;

m_h = one-way distance (ℓ_1-metric)from a collection unit to the disposal site, in miles;

m' = $(2m_h - 5.5)$, in miles;

N = designation for different policy on the maximum number of trips per day;

N_{max} = the maximum number of trips per day;

ΣN = total number of trips in a quarter (13 week period);

Σn_o = the number of days in a quarter in which a crew worked overtime;

$\left.\begin{array}{l} OT\ 1 \\ OT\ 2 \\ OT\ 3 \end{array}\right]$ = the amount of overtime permitted when servicing collection units, in hrs;

P = designation for different overtime policies;

Q_o = the percentage of collection units not receiving service;

Q_1 = the percentage of collection units receiving service on-time;

Q_2 = the percentage of collection units receiving service one day late;

Q_3 = the percentage of collection units receiving service one day early;

R = designation for different relay policies;

S = number of services, or living units;

T_h = haul time per day, in min;

t_d = time at the disposal site, per trip, including time spent in queue, in min per trip;

t_p = pickup time per trip, in min;

ΣT_d = total time at the disposal site in a quarter, in min;

ΣT_h = total haul time in a quarter; in min;

ΣT_o = total overtime in a quarter, in hrs;

ΣT_p = total pickup time in a quarter, in min;

$\overline{TV}$ = estimated time needed for activities other than loading, in min;

V = speed of travel, in mph;

α = a constant characteristic of the time needed to collect refuse from a service, in min per service;

γ = a constant characteristic of the time spent in traveling within the collection unit, in min per trip;

δ = a constant;

ε = a constant;

θ = pickup time for S services, in min;

$\overline{\theta}$ = the average pickup time for S services, in min;

μ = refuse production, in pounds per service per day;

ρ_s = service density, number of services per mile; and

σ_μ = standard deviation of the refuse production, in pounds per service per day.

CHAPTER 6

Extensions and Modifications of Simulation Models

The purpose of this chapter is to exercise the wisdom which grows out of hindsight. Experience with the simulation programs has indicated several areas where improvement should be sought. We shall list these and then discuss them.

1. Data base improvement.
2. Improvements in generating refuse variability.
3. Small loads and the capacity rule problem.
4. Data reduction, graphics and other computer-oriented improvements.

"Data base" is a buzz-word which has become fashionable in the late 1960's. It means, simply, the large amounts of systematized quantitative and qualitative information and the methods for keeping these current which must underlie any successful effort to apply system analytic methodologies. Efforts of the type conducted by the University of California[24] need to be repeated for current conditions in many places. Such studies should have the active participation of persons experienced in the design of experiments at an early stage, and as noted in Chapter 2, methods of regression analysis and analysis of variance and covariance, etc., should be applied to the data gathered to assess the relevance of different factors on refuse production and collection efficiency.

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[24] See Chapter 2 of Part 2.

Closely related to these ideas is the need for more definite knowledge of the probability laws that underlie refuse production (point 2 above). We have indicated on slim evidence that refuse production is normally distributed[25] in both simulation models. Disturbed by the possibility that we might generate negative amounts of refuse, we have truncated the left tail of the normal density at some arbitrary minimum refuse production. There may be probability laws that better describe refuse production and these should be sought.

Supposing that we have done this, and we have a clear idea how refuse production varies on the basis of pounds per service, per day, some valid questions may be raised as to how to turn this micro-analytic fact into a valid refuse production figure for a large number of services (a CU). We have relied on the central limit theorem of the theory of probability. This theorem states that the average value of a large number of observations tends to be normally distributed no matter what the probability law governing the individual observation. Thus, we have gotten a CU's refuse production by sampling from $N(D\mu, \sqrt{D}\,\sigma)$ where D is the number of days since the last collection. This value, if greater than the left-tail truncation value, is then multiplied by the number of services. One may very well ask, and be answered by embarrassed silence, why the refuse production wasn't generated form $N(SD\mu, \sqrt{SD}\sigma)$.[26] One may also note that it seems unlikely that on any given day one CU will be extremely high in refuse production while another will be low. We should seek, then, a scheme for generating refuse that causes groups of CU's of like nature to move together. One might also argue that refuse production may behave more like the Dow Jones Average than a Brownian particle, i.e., instead of up and down every day we expect several days of consecutive ups, some uncertain plateaus and some consecutive days of downs with occasional rallies.

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[25] Log normality has been observed in Raleigh, N.C. (Personal communication.)

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[26] This was done in the case of the simulation model in Raleigh, N.C., with the log normal density.

Closely allied to these problems are the need for more realistic ways of introducing seasonal variation in refuse production, the effects of weather on collection efficiency (and haul speed), and better knowledge of how haul speed varies both as a random variable and as a function of haul distance.

The ideas sketched in the previous paragraphs are important for the problem of small loads and capacity rules. We have noted previously that the introduction of some look ahead rationality and a very simple capacity rule produced fewer and larger small loads under the constant length work day rules where only two trips per day were permitted. Since the daily route method is still appropriate to many communities, we may ask if the proliferation of small loads, not observed to the same extent in practice in Winnetka as in simulation can be reduced by some means. The answer is yes: If groups of like CU's tend to go up and down together, if variation goes up and down in spells of several days, if collectors can read the calendar and see lawns beginning to sprout, then they can anticipate the amounts of refuse to be collected and base their capacity decision on a convenient number of loads of approximately equal size. Thus, if they anticipate a ten-ton day and the truck has a capacity of three tons, they aim for three loads of $3\frac{1}{3}$ tons each instead of two loads of 4 and one of 2 tons. This is probably much easier to state than to implement.

The last category of improvement is largely a matter of making better use of computer resources. Instead of making many runs and performing the necessary analysis by hand and slide rule as was done here, future programs will be written to store the results of many runs and then perform the analysis, and plot the graphs. Looking further ahead, but not too much, one may anticipate simulation programs where the investigator sits at a remote console cathode ray tube (CRT) and poses his questions and receives his visible output on the CRT in short order.

Mechanics, data and so forth settled, what sorts of problems in solid wastes should models of this sort be extended to? The answer is to every conceivable type of situation. We have considered two very simple situations. There are many more, and problems of growing importance in such areas such as supplemental transportation, methods of disposal, volume reduction, etc., can be furthered by the use of simulation.

CHAPTER 7

Summary and Conclusions

Simulation models were developed for a daily route method system of refuse collection based on the Village of Winnetka, Illinois, and for a constant length work day based on the City of Chicago. The development of these models is discussed in Chapter 3 and Appendix 2-A.

The daily route method model was exercised to delineate the interdependencies of several significant parameters involved in the functioning of a refuse collection system. In particular the model was used to find maximum, average and minimum value plots of

E_c = Percent of capacity utilized
N = Number of daily trips
E = Overall collection efficiency (man-min/ton)
H_w = Work day length (minutes)
E_h = Haul efficiency (man-min/ton-mile)
T_h/T = Percent of total time taken up by haul operation

against the coefficient of variability of refuse production, σ/μ , and the refuse assignment, A, the average daily amount of refuse in truckload units. The last four of the above list were also exhibited as functions (minimum, average, and maximum values) of m_h, the one-way haul distance (ℓ_1-metric). These graphs are supplemented by frequency histograms of truck capacity utilization (E_c) and length of workday (H_w) arranged by value of σ/μ and value of A . Also provided are plots that correlate system behavior (E) with changes in α and γ, the coefficients of the function which relates pickup time to the number of services (S) and the internal route mileage (m_i) . Finally, a plot is provided for a composite system of thirteen trucks to illustrate the behavior of the average and maximum queuing times at the disposal site as a function of the number of unloading docks available.

It is to be reiterated that the particular numerical values cited in the conclusions listed below pertain to the reference system described in sections A and B of Chapter 4. The values that would pertain to any other system would have to be derived by exercising the simulation model on data from that system.

1) Inspection of the frequency histograms of E_c shows that small partial loads are present at all values of σ/μ and A with a pronounced tendency to have a widely separated bimodal density at low values of both. Small loads are a function of the capacity rule and the demands of the daily route method as simulated in the present model.

2) The overall collection efficiency, E, is affected by changes in σ/μ . For the reference system, the daily route method does not seem to be desirable for values of $\sigma/\mu > 0.15$. This is the value of σ/μ for which the variation in the length of the work day amounts to approximately 30 minutes above and below the average.

3) Consideration of the frequency histograms of length of work day shows that the effect of σ/μ on the spread of the density are greatest for large values of A . We conclude that under the daily route method large A and large σ/μ should not go together.

4) For the reference system, increasing the assignment up to $A = 2$, particularly at low values of σ/μ increases capacity utilization by shifting upward and smoothing out the initially bimodal density. Above $A = 2$, the density continues to flatten, but not as dramatically. A is not the cause of small loads.

5) For the reference system, an assignment of $A = 1.5$ appears to provide an optimum time buffer in a system requiring 2-3 loads a day to effectively swallow the effects of random variation in refuse production. This results in essentially equal H_w for the minimum, maximum and average curves when $A = 1.5$. The underlying mechanism is unexplained.

6) An assignment of $A = 1.5$ also provides the overall best system performance in terms of constancy of number of loads per day and lowest collection efficiency.

7) The length of work day (H_w), overall collection efficiency (E) and proportion of haul time (T_h/T) are proportional to one-way haul distance, m_h. The haul efficiency is

essentially constant for $m_h = 4$ to 12 miles because distance is offset by greater haul speed.

8) For the reference system, a distance $m_h \leq 6$ miles is desirable in that the variation in the length of the workday does not exceed thirty minutes above or below the average for such **distances**.

9) The length of workday frequency histogram spreads out and becomes **bimodal** as the distance m_h increases. This reflects disparity between 3 and 4 day loads in a twice-a-week collection system.

10) The overall collection efficiency (E) responds to changes in α and γ that reflect the manner in which these are used in the program to determine pickup time.

11) Quarterly waiting time for a thirteen truck fleet is reduced exponentially by increases in the number of unloading docks. A fourth unloading dock saves 237 man-hours per year over three docks. A ratio of one dock to every three or four trucks appears to be indicated.

Alternate policies of last load relay, overtime and assignment (time assignment and refuse assignment) were tested for twelve of twenty-four possible combinations in a simulation of a constant length workday collection system. The significant findings are

12) Unit cost with units dollars per living unit per week are a more accurate index of the effect of a policy as the number of living units serviced per week is essentially constant while unit cost based on tons of refuse fluctuates due to variable refuse production. The use of the former unit cost is indicated and used below.

13) The use of a relay driver to haul the last load of the day generally improves the quality of service as measured by on-time collection. This is bought at an increased unit cost.

14) When relay and overtime are used together too much slack may be provided for the shocks of variation. This may result in ahead-of-time collections (not always beneficial).

15) Decreasing the assignment results in better service at a higher unit cost. The law of

diminishing returns is clearly at work here with earlier decreases in the assignment more cost effective than later ones.

16) The line $A_t = 1$ (where A_t is the expected daily time assignment in 8-hour units) provides a dividing line on the cost effectiveness of overtime. At all levels of A_t, some improvement in quality of service is found with the use of overtime. For $A_t > 1$ this improvement results in increased unit cost; for $A_t < 1$, the improvements in quality results in essentially no increase in unit cost.

17) Overtime cannot wholly compensate for too large a refuse assignment A_r. Good service at low unit cost is secured with A_r = approximately 2 and A_t less than 1.

APPENDIX 2-A

I. Introduction

The purpose of this appendix is to provide a detailed manual for the understanding, use, and eventual further extension of the system of programs that has been written to simulate the operations of a rufuse collection and haul system.

Some of the general material appearing in Section II below has already appeared in Chapter 2, Section 3. We repeat it here in a more succinct form to allow all material pertaining to the daily route simulation model to appear in one place.

II. An Overview of Concepts Used in this Program

1. General Description

A. Physical Organization of the City: For the purposes of this simulation, the city is divided into "districts." The district is meant to convey characteristics of neighborhood type, and as such, a given district need not be physically contiguous. Such characteristics of neighborhood type, important to the design of a refuse collection system, might include presence or absence of alleys, one and two-family or multiple-family dwellings, income, size of families, etc. The data to be provided for each district are:

a) Average refuse production in pounds per service per day.

b) The standard deviation of same.

c) The parameters of a function relating refuse pickup time to the number of services, the amount of refuse, service density, and internal mileage of sub-areas or some combination of some or all of these factors.

The data provided in a), b), and c) are applicable homogeneously in all subdivisions of the district. The simulation system allows up to 10 districts.

Each district is further subdivided into "collection units" (CU's henceforth) which are small, convenient physically contiguous areas lying wholly within their respective districts. A given CU is serviced by only one truck.

The data to be provided for each CU includes:

a) An indication of which days of the week it is to be serviced.

b) Which district it belongs to.

c) The number of services.

d) Service density.

e) Its physical location.

The data provided in a) - e) are applicable homogeneously in all portions of a CU . While there is no named entity smaller than a CU, this assumption of homogeneity is important when a truck does not finish a CU before returning to the disposal site, in which case a partial CU remains to be serviced. The simulation system provides for a maximum of 1000 CU's in the entire city.

A single disposal site and a garage are provided for. They may be at different physical locations.

The physical locations of the CU's, the garage, and the disposal site are given by their Cartesian coordinates. The origin of the coordinate system is immaterial, but must be internally consistent throughout a problem.

B. Method of Work Organization: This simulation system simulates refuse collection according to the daily route method. Under this method of work organization, each crew is assigned one or more work areas (zones, beats) which are to be serviced some fixed number of times per week. The area assigned for any given day of the week must be completely serviced on that day. Each crew operates independently, helped by and helping no other crew and quitting early or late as the work load demands.

Each truck (crew) is given a string of consecutively numbered CU's. Within this string, those to be serviced on any given day do not have to be adjacent, but day-mate CU's should be arranged so that the higher numbered CU's are progressively nearer the disposal site.

The data to be provided for each truck are:

a) The number of the first CU and the total number of CU's in its zone.

b) Truck capacity.

c) Crew size.

The simulation provides for up to 64 trucks to be operating simultaneously.

C. Time Organization: The largest time period recognized by the simulation system is a World Calendar quarter of 91 days (13 weeks). The quarter is the basic reporting period of the simulation system; data accumulated on a trip or daily basis is printed and/or punched out for each truck at the end of each simulated quarter.

The next largest time unit is the month. A World Calendar quarter consists of three months of 31, 30 and 30 days in that order. Weather induced slowdowns and seasonal adjustments in refuse production are changed on a monthly basis.

The customary 7-day week is the basis for scheduling the work of the trucks. A given CU may be serviced from 1 to 6 times per week.

The actual simulation proceeds on a daily basis governed by an outer program regulating the counting of days into weeks, months and quarters. The day begins with the dispatching of each truck that is to work on the given day of the week to its first CU of the day and ends when all trucks have returned to the garage.

Within a day are variable length entities called "events" of minimal length zero. The day's operation of a truck is composed of a string of events: leave garage, arrive at first CU, finish first CU, ..., finish load, arrive at disposal site, leave disposal site, ..., arrive at garage. The setting of events is governed by the string of events that has already occurred. The sequencing of events is governed by a section of the main program called the time status record (TSR or event clock). A trip is coincidental with the event: "truck arrives at disposal site with refuse."

Closely related to events is an entity called status. An event tells the program what a truck will do do next; status tells what the truck is now doing. Status is not used in the simulation model at present, but is included to facilitate future inclusion of (a) truck breakdown event(s). The interpretation of the status code is given in the Glossary.

D. On Some Computational Methods and Assumptions: While most of the following information will be repeated later in greater detail, a general description of the simulation system requires a few general remarks about some of the basic determinants of the simulated results.

(a) Determination of Refuse Amounts: Refuse production is assumed to be normally distributed with the left tail of the normal curve truncated at 1 pound per service for any CU. The mean and standard deviation of refuse production given for the CU's district are modified according to the number of days since the last collection by the following scheme:

Let μ and σ be the mean and standard deviation respectively of refuse production in a given CU (pounds per service per day);

Let D be the number of days since this CU was last serviced;

Let S be the number of services in this CU;

And let F be the seasonal factor pertaining to the current month.

Then, the refuse (in tons) to be collected from this CU is given by a random variable R where

$$R \sim N[\mu \cdot D, \sigma \cdot \sqrt{D}] \cdot S \cdot F/2000$$

(See Figure 2-4 and page 2-67 .)

(b) Determination of pickup time: Linear regression analysis of 43 trips in Winnetka, Illinois (March-April, 1958) suggests that the pickup time is governed by the number of services; the effect of the amount of refuse is statistically insignificant. The regression equation is

$$\text{Time (minutes)} = \alpha S + \gamma$$

where S = number of services

We have modified this formula on the assumption that γ is at least partially explained by mileage travelled during collection. Thus, the formula used in the simulation system is

$$\text{Time (minutes)} = \alpha S + \gamma' m_1$$

where m_1 = internal mileage

$\gamma' = \gamma/\beta$

β = mean internal mileage of the CU's of the regression analysis.

(c) Determination of distances: The computation of distances is based on the so-called "metropolitan metric" (ℓ_1-metric) in which the path from one point to another is an L-shaped path:

$$d(p, p') = |x - x'| + |y - y'|$$

where $d(p, p')$ = distance from point $p = (x, y)$ to point $p' = (x', y')$ and "$|\ |$" = absolute value.

The use of this metric is justified by the rectangular grid arrangment of many North American street plans.

(d) Determination of speeds of travel times: The speed of trucks during their hauls is determined as a function of the distance to be traveled as computed above. The shape of the curve relating speed to distance is shown by Figure A.1. The equations for this curve are also indicated therein. These equations are based on a least squares fit of the curve given in Figure 29 of "An Analysis of Refuse Collection and Sanitary Landfill Disposal," University of California, 1952.

(e) The Effect of Weather and Seasonal Variation: At the beginning of each day, a weather severity factor is generated from a frequency distribution àpropos to the month of operation. This weather severity factor

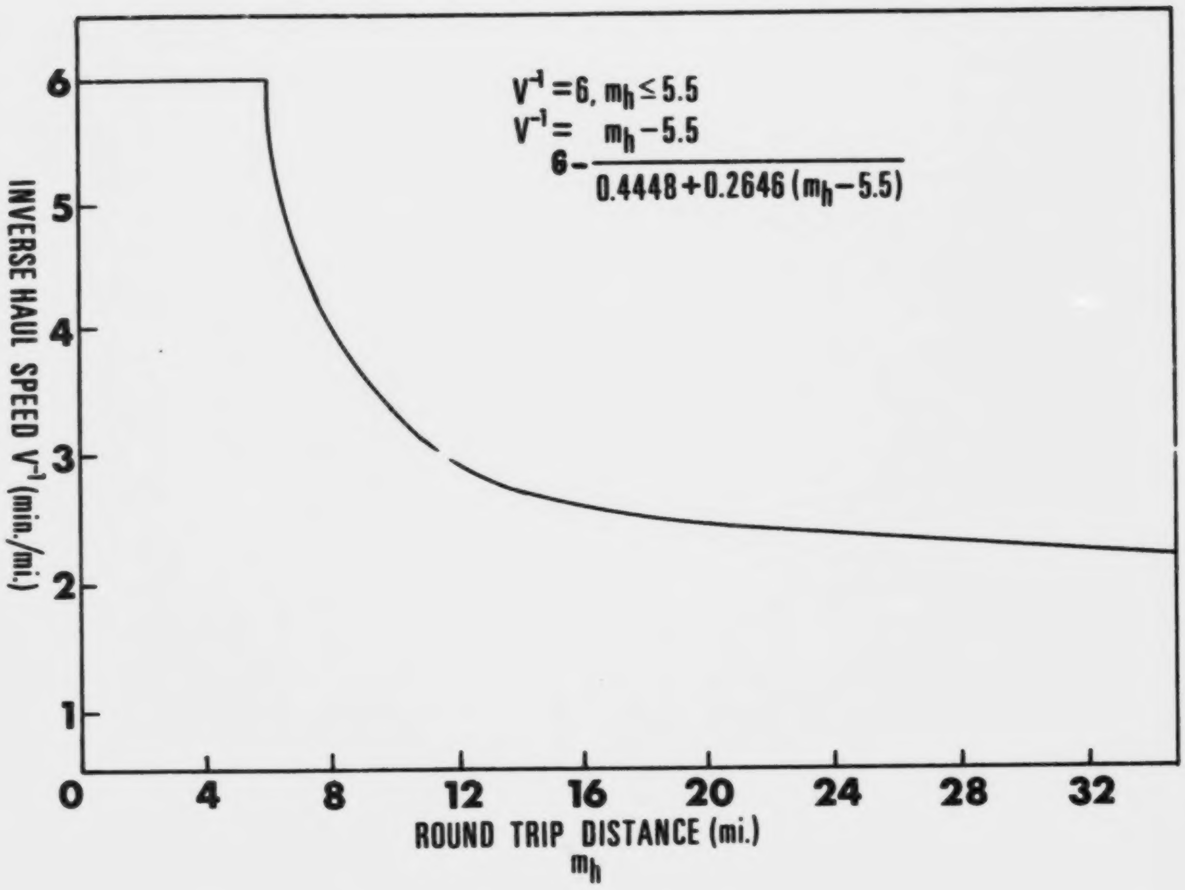


Figure A.1 Haul Speed vs. Round Trip Distance

is multiplied into the collection time for each collection unit serviced during the day. A weather severity factor of 1 means that today's weather is normal; greater than 1 implies a weather-induced slowdown.

Seasonal variation in refuse production is likewise handled by a multiplicative factor. Taking some month, say January, as the base month, the seasonal factors of the other months are the ratios of average refuse production in those months to the average refuse production in January. No provision is made to alter the variability of refuse production seasonally, and the seasonal factor is the same for all districts and CU's.

III. Detailed Description of the Simulation System

The simulation system consists of a main program, RCS002 (Refuse Collection Simulation), and a series of subroutines. Most of these subroutines are called only by the main program, a few are called by both the main program and one or more subroutines, while others are called only by other subroutines.

The main program contains the basic structure of the simulation and consists of three sections: the outer program which reads in the input data, passes time by days, weeks, months, and quarters, presets the proper initial conditions peculiar to each time period, and calls for the quarterly output of results; the event clock or time status record which sequences the events of all operating trucks and passes time within the daily simulation; and finally, the events themselves which are the basic representations of the movements and operations of the trucks, and which give rise to the occurrence of further events to be processed by the event clock. In the following description, the subroutine "PUT" which is an integral part of the event clock, is described with the event clock rather than with the subroutines.

The subroutines are divided into three groups: data generating subroutines, two in number, which generate refuse amounts, collection times, pickup mileages, haul times and haul distances; auxiliary subroutines which perform such functions as determining when a truck is full, finding the next CU, accumulating statistics, etc; and miscellaneous subroutines which generate random numbers, round off decimals, etc.

In the following descriptions, flowcharts of two types are presented. The first type is a verbal flowchart avoiding the use of subscripts and actual program terminology and giving a general understanding of the purposes and the functions of the program section under discussion. The second type is a detailed computer program flowchart.

1. The Main Program (RCS002)

A. The Outer Program: The outer program performs the following functions:

a) Reads input data, checks them for internal consistency and stores them where they will be needed.

b) Sets up and keeps track of the various time counters (quarters, months, weeks, days) and takes the following appropriate actions:

(1) End of quarter: Prints quarterly results and returns either to seek new data, terminate all runs, or to set up conditions for the next quarter's simulation. The latter consists of setting monthly, weekly, and daily time counters to their initial values, and initializing all quarterly output data accumulators.

(2) End of week: Sets day of week counter (TODAY) back to the number of the first day of the week on which any truck works.

(3) Beginning of day: Sets up first job for each truck that is working today and selects today's random weather event.

(4) End of day: Advances day counter and all other time counters to check for the end of the larger time periods.

(5) Beginning of month: Selects new monthly seasonal factor.

The following verbal flowchart (Figure A.2) outlines the actions of the outer program. The event clock and the events are each represented by a single box which will be expanded below.

The variable names used in the detailed flowchart are explained in the Glossary of this Appendix.

B. The Event Clock and "PUT" Subroutines: As discussed above, the day starts with a group of trucks leaving their garage at the same time. As the day progresses, differing refuse amounts, work rates and distances, etc. quickly put an end to synchronism between trucks. Thus, an orderly procedure is required to keep track of what each truck is doing. This will be all the more important as further areas of truck-to-truck interaction are added to the program (e.g. relay methods).

The event clock is a section of the main program which utilizes a set of lists called collectively the "time status record" (TSR) to properly sequence pending events and cause the passage of time.

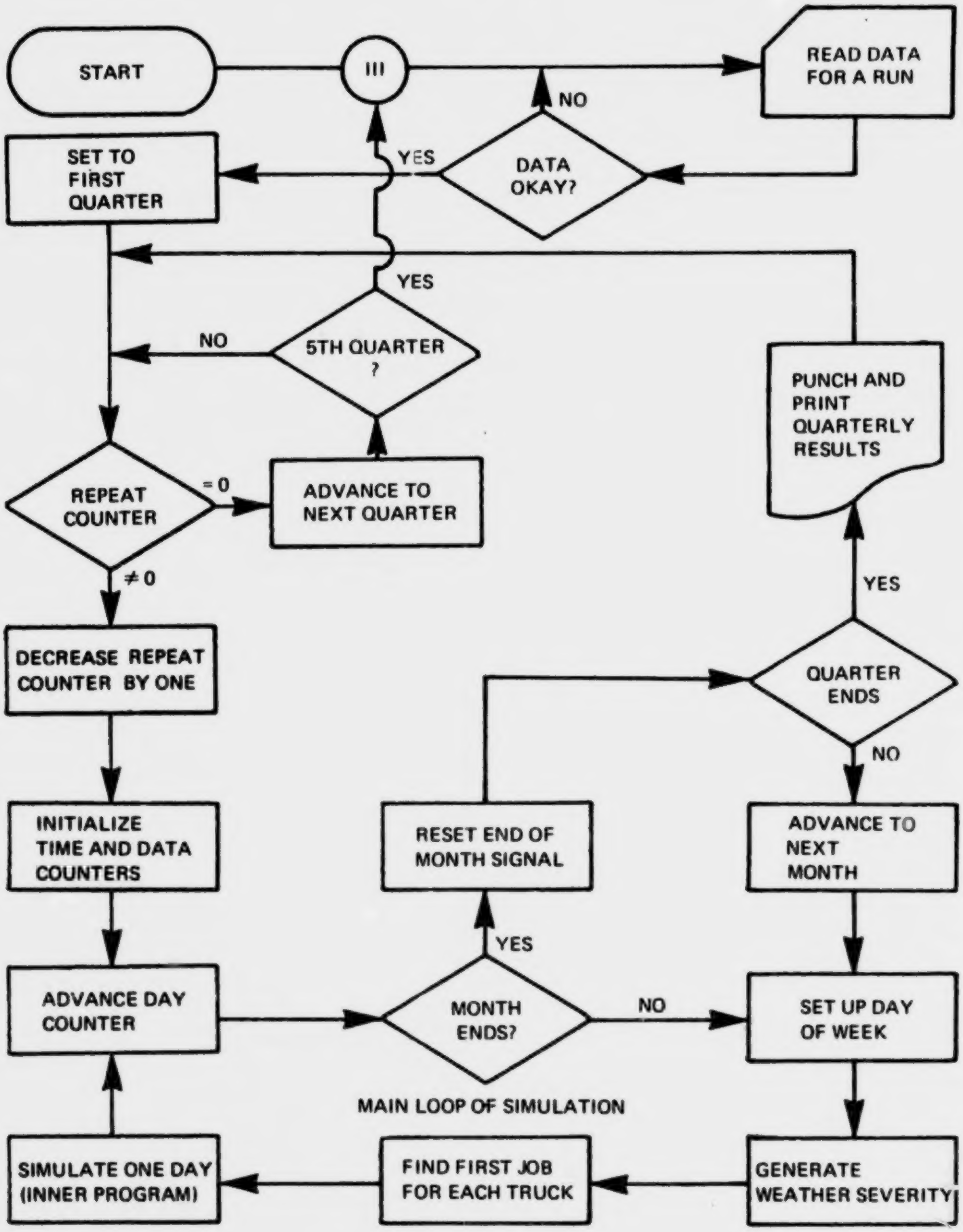

START
III
READ DATA FOR A RUN
NO
YES
DATA OKAY?
SET TO FIRST QUARTER
YES
NO
5TH QUARTER ?
PUNCH AND PRINT QUARTERLY RESULTS
REPEAT COUNTER
= 0
ADVANCE TO NEXT QUARTER
≠ 0
YES
DECREASE REPEAT COUNTER BY ONE
QUARTER ENDS
NO
INITIALIZE TIME AND DATA COUNTERS
RESET END OF MONTH SIGNAL
ADVANCE TO NEXT MONTH
YES
ADVANCE DAY COUNTER
MONTH ENDS?
NO
SET UP DAY OF WEEK
MAIN LOOP OF SIMULATION
SIMULATE ONE DAY (INNER PROGRAM)
FIND FIRST JOB FOR EACH TRUCK
GENERATE WEATHER SEVERITY

Figure A.2 Verbal Flowchart of Outer Program

In its simplest form, the TSR consists of two lists, a list of pending events (PEVLST) and a list of the time of their occurrence (TIMLST), the latter in monotonic non-decreasing order. By "event" is meant a subroutine or as in this program a section of the main program that simulates a certain kind of system action and through whose action the future occurrence of other events may be initiated. By "time of occurrence" is meant the time from "right now" which must pass until the event will occur.

The TSR used in this program differs only in that a third list is carried (PTRNO) which specifies the number of the truck which will be involved in the pending event. The operation of the simulation through the event clock takes place as follows (see Figure A.3).

1) The number of events counter (NOEV) is tested. If NOEV = 0, the simulation ends; otherwise go to step 2 .
2) Pick up the first occurring pending event and its associated time and truck number. By the ordering of the times, these will be the first items on their respective lists.

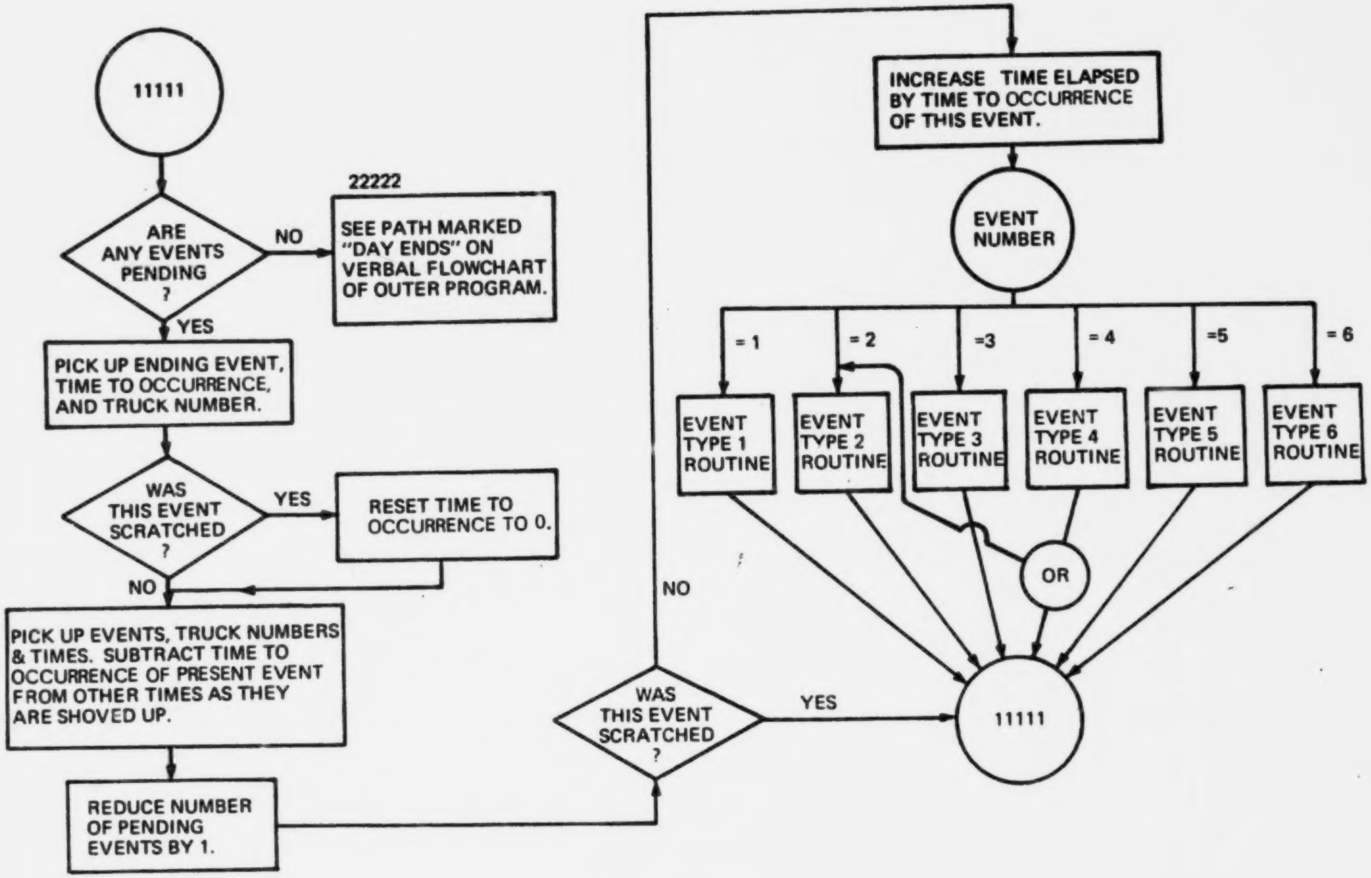

11111
ARE ANY EVENTS PENDING ?
NO
22222
SEE PATH MARKED "DAY ENDS" ON VERBAL FLOWCHART OF OUTER PROGRAM.
YES
PICK UP ENDING EVENT, TIME TO OCCURRENCE, AND TRUCK NUMBER.
WAS THIS EVENT SCRATCHED ?
YES
RESET TIME TO OCCURRENCE TO 0.
NO
PICK UP EVENTS, TRUCK NUMBERS & TIMES. SUBTRACT TIME TO OCCURRENCE OF PRESENT EVENT FROM OTHER TIMES AS THEY ARE SHOVED UP.
REDUCE NUMBER OF PENDING EVENTS BY 1.
WAS THIS EVENT SCRATCHED ?
NO
YES
INCREASE TIME ELAPSED BY TIME TO OCCURRENCE OF THIS EVENT.
EVENT NUMBER
= 1
= 2
=3
= 4
=5
= 6
EVENT TYPE 1 ROUTINE
EVENT TYPE 2 ROUTINE
EVENT TYPE 3 ROUTINE
EVENT TYPE 4 ROUTINE
EVENT TYPE 5 ROUTINE
EVENT TYPE 6 ROUTINE
OR
11111

Figure A.3 Verbal Flowchart of Event Clock

3) If the event has been scratched[27/] (in the sense of horse-racing) the incremental time is set to 0; otherwise the incremental time is the time associated with the event in step 2 .

4) All lists are shoved up one place, i.e., 2nd → 1st, 3rd → 2nd, etc., and simultaneously each time is reduced by the incremental time as determined in steps 2 and 3 .

5) The number of events counter (NOEV) is reduced by 1.

6) If the event has been scratched, return to step 1 ; otherwise go to step 7 .

7) The master daily time counter (TELAPS) is increased by the incremental time.

8) The appropriate event program is entered.

With one exception (see Event Type 4, below) all event programs terminated by returning to step 1 of the event clock.

When one of these events generates the occurrence of a future event, this event together with its truck number and time of occurrence are entered into the TSR by the PUT subroutine which is called as: CALL PUT (TIME,

---

[27/] See Event Type 6 below.

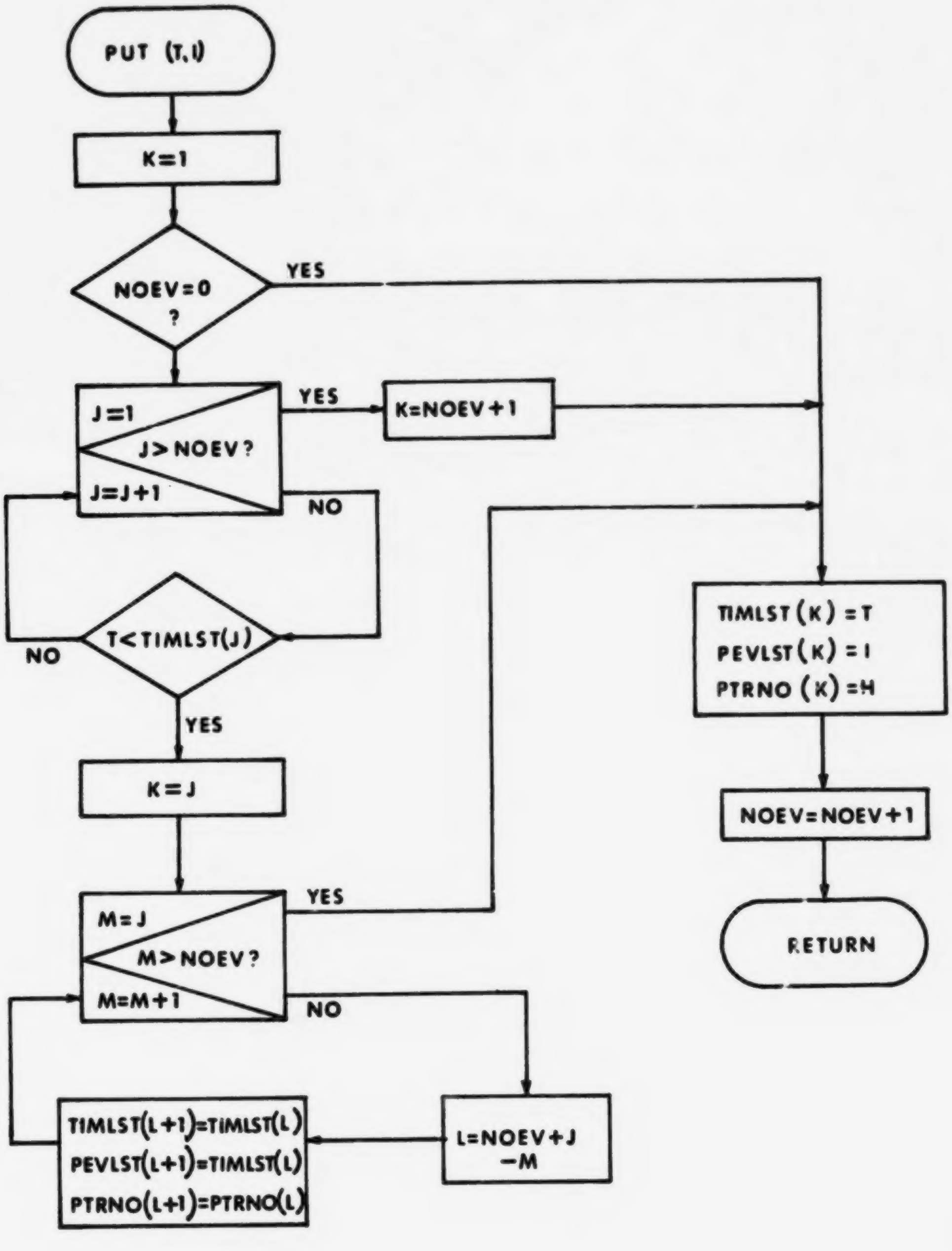
EVENT) (the truck number is carried in a COMMON section).

The PUT subroutine (see Figure A.4) using the counter NOEV searches down TIMLST for the first entry thereof which exceeds "TIME". If all entries of TIMLST $\leq$ TIME, then TIME, EVENT and truck number are placed beyond the end of TIMLST, PEVLST and PTRNO, respectively and NOEV + 1 replaces NOEV. Otherwise, denoting K $\leq$ NOEV as the index of the first entry such that TIMLST(k) > TIME, the entries of all three lists are moved up (NOEV - th → NOEV + 1 - st, ... Kth → K + 1 - st) and the new entries are inserted at the Kth position and finally NOEV + 1 replaces NOEV.

The present version of the program allows for the stacking of 500 pending events.

C. The Events: The events described hereinunder are called event type 1, 2, etc. This numbering has no relation to their typical order of occurrence, but merely reflects the historical accident of the order in which they were written.

These program segments are best understood in terms of their detailed flowcharts (A.5 - A.10) and the Glossary of Notation. We give here verbal descriptions with a minimum of frills and terminology.



PUT (T,I)
K=1
NOEV=0 ?
YES
J=1
J>NOEV?
J=J+1
YES
K=NOEV+1
NO
T<TIMLST(J)
NO
YES
K=J
M=J
M>NOEV?
M=M+1
YES
NO
TIMLST(L+1)=TIMLST(L)
PEVLST(L+1)=TIMLST(L)
PTRNO(L+1)=PTRNO(L)
L=NOEV+J -M
TIMLST(K)=T
PEVLST(K)=I
PTRNO(K)=H
NOEV=NOEV+1
RETURN

Figure A.4 PUT-Subroutine for Storing Events

a) Event Type 1 ... Truck Arrives at Disposal Site (Figure A.5): At the beginning of the program a logical variable (SKPDK) is set to .TRUE. if the number of unloading docks NLD $\geq$ the number of trucks and to .FALSE. otherwise. This is used to shunt around a great deal of the logic in this section if no unloading bottleneck is possible.

1) Set the time at the disposal site counter for this particular truck to minus the value of the master daily time counter (TELAPS). This is done so that when the truck leaves the disposal sites (Event Type 3, below), the time at the D.S. can be gotten simply by adding the value of TELAPS at that time.

2) If SKPDK = .TRUE. go to step 8 . If SKPDK , .FALSE., go to step 3 .

3) Locate an unused unloading dock. If one can be found, denote its number by II and go to step 8 . If all unloading docks are busy, go to step 4 .

4) Locate the first occurring event type 3 in the TSR. This tells when some other truck will leave the disposal site freeing an unloading dock. Call the associated time TIMLST(IJ).

5) PUT a new event type 1 in the TSR at TIMLST(IJ). This forces the truck to arrive again at

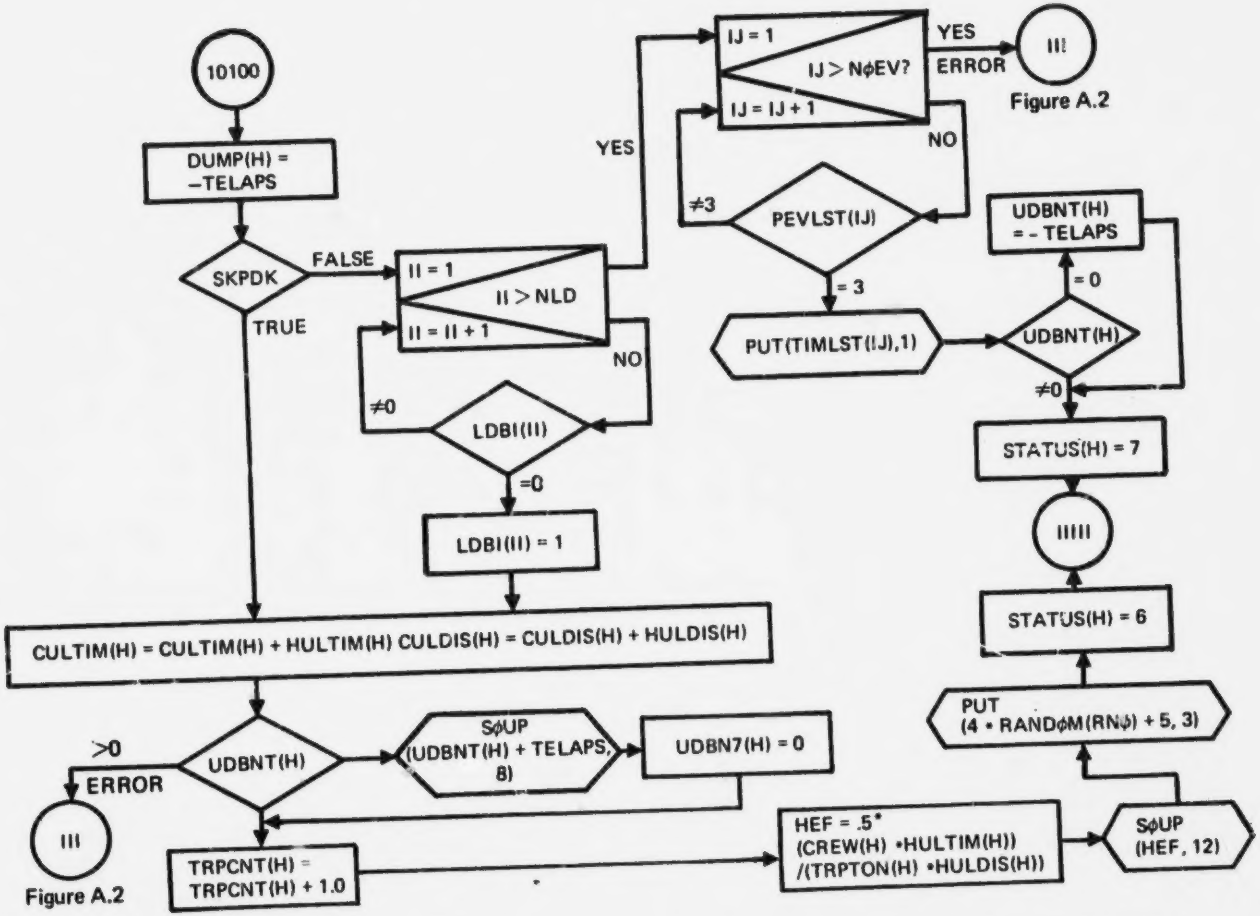

10100
DUMP(H) = −TELAPS
SKPDK
FALSE
TRUE
II = 1
II > NLD
II = II + 1
YES
NO
LDBI(II)
≠0
=0
LDBI(II) = 1
IJ = 1
IJ > NϕEV?
IJ = IJ + 1
YES
ERROR
II!
Figure A.2
NO
PEVLST(IJ)
≠3
= 3
PUT(TIMLST(IJ),1)
UDBNT(H)
= 0
UDBNT(H) = - TELAPS
≠0
STATUS(H) = 7
IIIII
STATUS(H) = 6
PUT (4 • RANDϕM(RNϕ) + 5, 3)
CULTIM(H) = CULTIM(H) + HULTIM(H) CULDIS(H) = CULDIS(H) + HULDIS(H)
UDBNT(H)
>0
ERROR
III
Figure A.2
SϕUP (UDBNT(H) + TELAPS, 8)
UDBN7(H) = 0
TRPCNT(H) = TRPCNT(H) + 1.0
HEF = .5* (CREW(H) •HULTIM(H)) /(TRPTON(H) •HULDIS(H))
SϕUP (HEF, 12)

Figure A.5 Event Type 1: Truck Arrives at Disposal Site

the disposal site; in effect, the truck is placed on a queue waiting for a free unloading dock.

6) If this truck was arriving at the disposal site fresh (i.e., not on the queue) then its unloading dock bottleneck time counter UDBNT (truck #) will be zero. At this point, if UDBNT(truck #)= 0, set UDBNT(truck #) = -TELAPS(see remark in step 1 above), otherwise leave it as is.

7) Go to step 1 of the event clock.

8) Set unloading dock #II to busy status.

8´) Accumulate the haul time and haul distance generated in reaching the disposal site.

9) If UDBNT(truck #) = 0, go to step 11 . If < 0, go to step 10 .

10) Compute unloading bottleneck time = UDBNT (truck #) + TELAPS and generate statistics (see subroutine SOUP). Reset UDBNT (truck #) to zero.

11) Accumulate statistics (SOUP) on haul efficiency (man • minutes per ton • mile). Increase trip counter (truck #) by 1.

12) Generate a random unloading time uniformly distributed between 5 and 9 minutes and PUT an event type 3 in the TSR to occur at this time.

13) Go to step 1) of the event clock.

b) Event Type 2 ... Collection Unit Entered for First Time (Figure A.6): The net effect of this event is to set up the job to be done on a fresh CU and to determine how much can be taken aboard the truck (the truck may have serviced other CU's before starting this one). This event sets up the job continuation with an event type 4.

1) Accumulate the haul time and haul distance that were generated in reaching this CU.

2) Call subroutine CMPJOB(compute job) to generate the random amount of refuse (RFPCU) in this CU, the time to collect this refuse (TTFPCU), and the internal distance (DINPCU) to be traveled while collecting. Each of these quantities is indexed by truck #.

3) Call subroutine CAPRUL (capacity rule) to generate the proportion PROP of the refuse present in this CU (PROP is indexed by truck #) that can be loaded.

4) PUT an event type 4 (CU is partially finished) in the TSR to occur at time = PROP × TTFPCU.

5) Go to step 1 of the event clock.

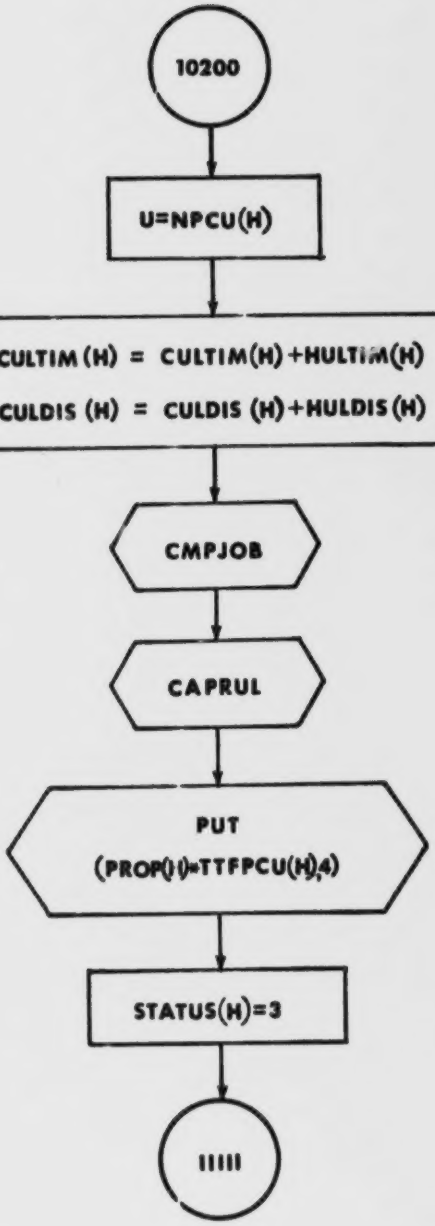

10200
U=NPCU(H)
CULTIM (H) = CULTIM(H)+HULTIM(H)
CULDIS (H) = CULDIS (H)+HULDIS (H)
CMPJOB
CAPRUL
PUT
(PROP(1)=TTFPCU(H),4)
STATUS(H)=3
11111

Figure A.6 Event Type 2: Collection Unit Entered for First Time

c) Event Type 3 ... Truck Leaves Disposal Site (Figure A.7). The function of this event is to accumulate trip statistics and to continue the job either

- i) Re-entering an unfinished CU
- ii) Going to an unstarted CU
- iii) Going to the garage (all CU's serviced.

1) If SKPDK (see Event Type 1, above) = .TRUE. go to step 3 . Otherwise go to step 2 .

2) Set an unloading dock to not-busy status.

3) Compute proportion of capacity utilized = Tonnage of refuse aboard / capacity of truck in tons and update histogram.

4) Accumulate statistics on trip counters of tonnage, time, and distance by adding in trip counters thereof.

5) Accumulate statistics on trip counters of tonnage, time, and distance (see SOUP subroutine).

6) Bring time spent at disposal site counter up-to-date for this truck by adding TELAPS to it (see Event Type 1, step 1), and accumulate statistics (SOUP).

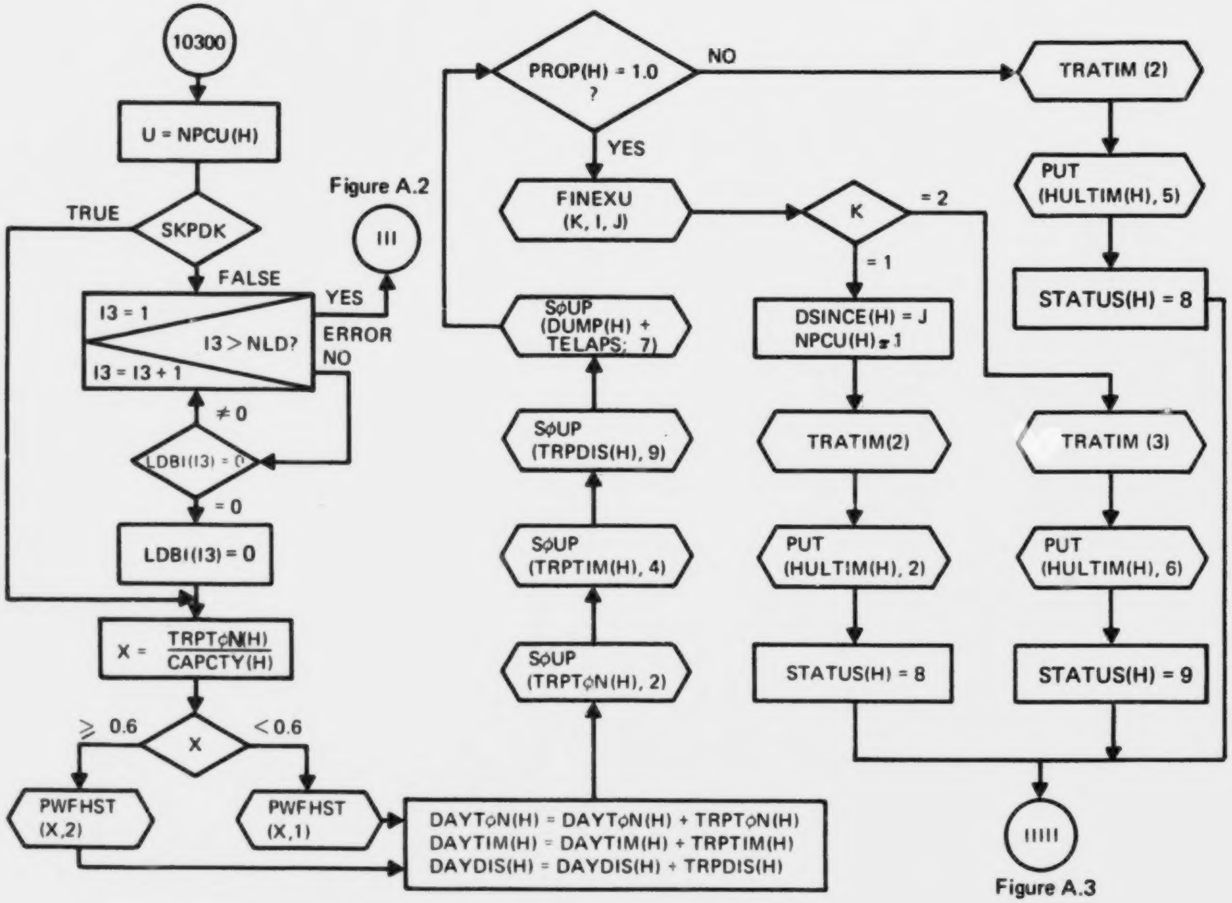

10300
U = NPCU(H)
SKPDK
TRUE
FALSE
I3 = 1
I3 > NLD?
I3 = I3 + 1
YES
ERROR
NO
Figure A.2
III
≠ 0
LDBI(I3) = 0
= 0
LDBI(I3) = 0
X = TRPTφN(H) / CAPCTY(H)
X
≥ 0.6
< 0.6
PWFHST (X,2)
PWFHST (X,1)
DAYTφN(H) = DAYTφN(H) + TRPTφN(H)
DAYTIM(H) = DAYTIM(H) + TRPTIM(H)
DAYDIS(H) = DAYDIS(H) + TRPDIS(H)
SφUP (TRPTφN(H), 2)
SφUP (TRPTIM(H), 4)
SφUP (TRPDIS(H), 9)
SφUP (DUMP(H) + TELAPS; 7)
PROP(H) = 1.0 ?
NO
YES
FINEXU (K, I, J)
K
= 2
= 1
DSINCE(H) = J
NPCU(H) = 1
TRATIM(2)
PUT (HULTIM(H), 2)
STATUS(H) = 8
TRATIM (2)
PUT (HULTIM(H), 5)
STATUS(H) = 8
TRATIM (3)
PUT (HULTIM(H), 6)
STATUS(H) = 9
IIIII
Figure A.3

Figure A.7 Event Type 3: Truck Leaves Disposal Site

7) If the proportion completed at the last CU serviced by this truck (PROP) is less than 1, go to step 8 , otherwise go to Step 11 .

8) Entering here indicates there is a CU which has not been completely serviced. Compute the haul distance and haul time from the disposal site to this CU (subroutine TRATIM).

9) PUT an event type 5 (partially done CU entered) into the TSR at time = haul time just computed.

10) Go to step 1 of the event clock.

11) Determine whether truck is through for the day. This is done with subroutine FINEXU (find next unit) and is indicated by an inability to find a successor unit. If the truck is through for the day, go to step 12 , otherwise go to step 15 .

12) Compute the haul distance and haul time from the disposal site to the garage (TRATIM).

13) PUT an event type 6 (truck arrives at garage) into the TSR at time = haul time just computed.

14) Go to step 1 of the event clock.

15) If the truck is not through for the day: Compute the haul time and haul distance (TRATIM) from the disposal site to the next CU found by FINEXU.

16) PUT an event type 2 (collection unit entered for first time) into the TSR at time = haul time just computed.

17) Go to step 1 of the event clock.

d) Event Type 4 ... Collection Unit is Partially Finished (Figure A.8). This event is the successor to the two CU entry events (2 above and 5 below). In this event trip tonnage, time, and distance measurements are accumulated, and it is determined whether the truck must now go to the disposal site or whether it has enough empty space to permit it to go on to the next CU.

1) Accumulate trip tonnage, and distance measurements.

2) If the proportion (PROP) of this CU collected is less than 1, go to step 3 , otherwise go to step 6 .

3) Compute haul time and haul distance from this CU to the disposal site (subroutine TRATIM).

4) PUT an event type 1 (truck arrives at disposal site) into the TSR at time = haul time just computed.

5) Go to step 1) of the event clock.

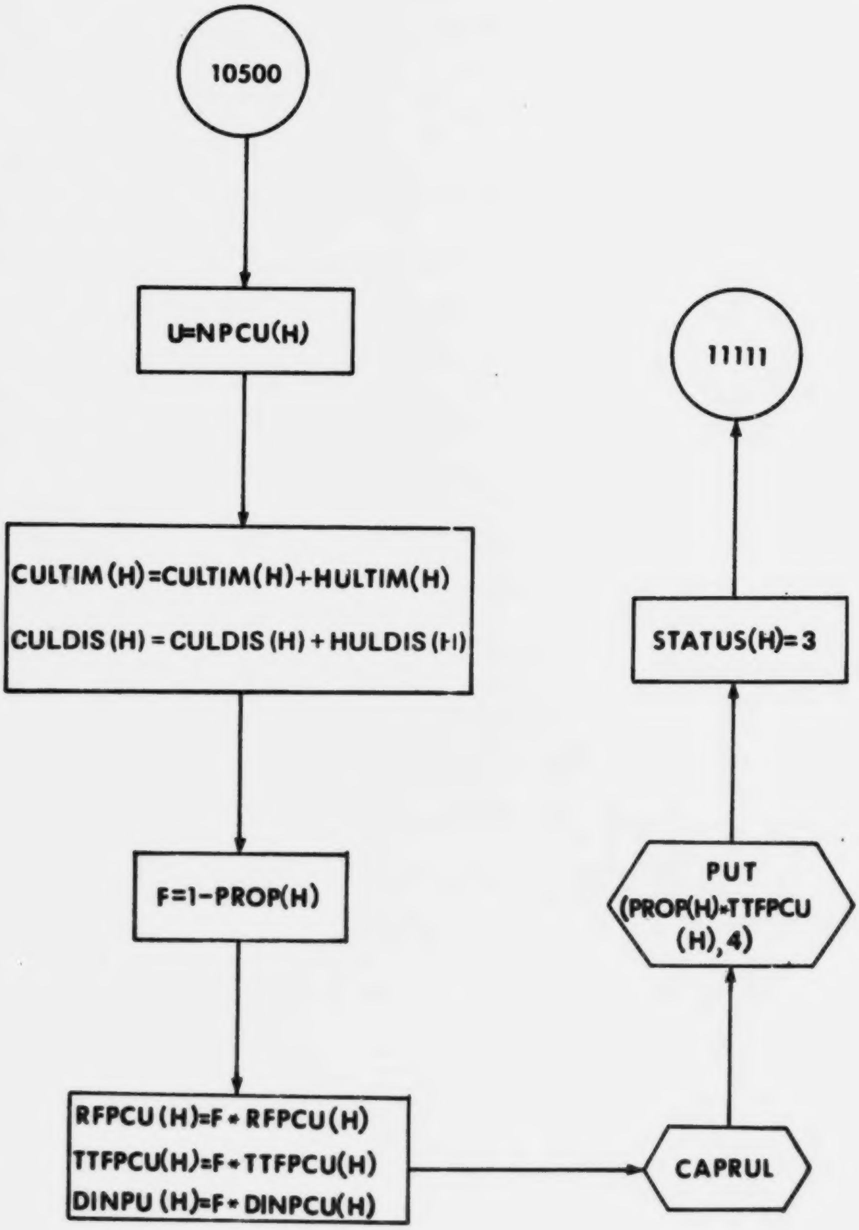

10500
U=NPCU(H)
CULTIM (H)=CULTIM(H)+HULTIM(H)
CULDIS (H) = CULDIS (H) + HULDIS (H)
F=1-PROP(H)
RFPCU (H)=F * RFPCU(H)
TTFPCU(H)=F * TTFPCU(H)
DINPU (H)=F * DINPCU(H)
CAPRUL
PUT (PROP(H)*TTFPCU (H), 4)
STATUS(H)=3
11111

Figure A.8 Event Type 4: Collection Unit is PROP(H) Finished

6) Determine whether truck is through for the day (see step 11 in event type 3 above). If yes, go to step 3 above, otherwise go to step 7.

7) Set up parameters for going directly to the next CU and transfer directly to step 1 of event type 2 without going through the event clock. *This assumes that there is negligible haul time and distance between successive CU's of a truck's route*.

e) *Event Type 5 ... Partially Done CU is Entered* (Figure A.9). This event occurs only as a result of a truck returning to collection after a visit to the disposal site, but it is not the only event that may succeed such a visit.

1) Accumulate the haul time and haul distance generated in reaching this CU from the disposal site.

2) Let F = 1 - PROP where PROP = the proportion of this CU that has already been serviced. Then, let RFPCU × F replace RFPCU = refuse present CU; TTFPCU × F replace TTFPCU = time to finish present CU; DINPCU × F replace DINPCU = distance inside present CU. These latter quantities having been computed originally in event type 2 by subroutine CMPJOB.

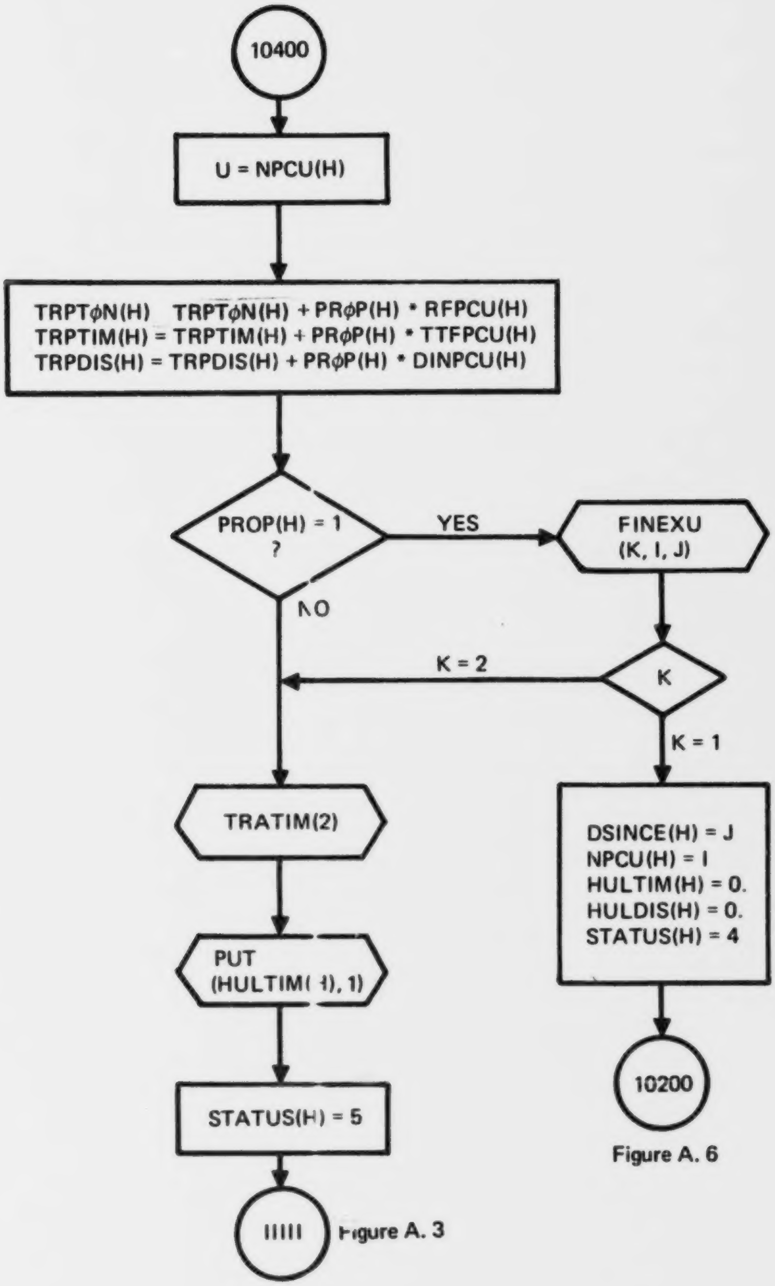

10400
U = NPCU(H)
TRPTφN(H) TRPTφN(H) + PRφP(H) * RFPCU(H)
TRPTIM(H) = TRPTIM(H) + PRφP(H) * TTFPCU(H)
TRPDIS(H) = TRPDIS(H) + PRφP(H) * DINPCU(H)
PROP(H) = 1 ?
YES
NO
FINEXU (K, I, J)
K
K = 2
K = 1
TRATIM(2)
PUT (HULTIM(H), 1)
STATUS(H) = 5
DSINCE(H) = J
NPCU(H) = I
HULTIM(H) = 0.
HULDIS(H) = 0.
STATUS(H) = 4
10200
Figure A. 6
11111
Figure A. 3

Figure A.9 Event Type 5: Partially Done Collection Unit is Entered

3) Enter subroutine CAPRUL (capacity rule) to determine proportion (PROP) of remainder thus computed that will now fit into empty truck.

4) PUT an event type 4 (Collection Unit is Partially Finished) into TSR at time = PROP × TTFPCU.

5) Go to step 1) of the event clock.

f) Event Type 6 ... Truck Arrives at Garage (Figure A.10). This event is the termination of the day's work for each truck. Statistics on daily tonnage, time, and distance are accumulated and a histogram record of hours worked is updated. Any pending events pertaining to this truck are scratched. This latter task is superfluous in the present program, but is included for the sake of flexibility.

1) Accumulate statistics on daily tonnage, time, and distance, and on haul time, haul distance, and trip count.

2) Update hours-worked histogram.

3) Scratch any pending events in TSR pertaining to this truck.

4) Go to step 1 of the event clock.

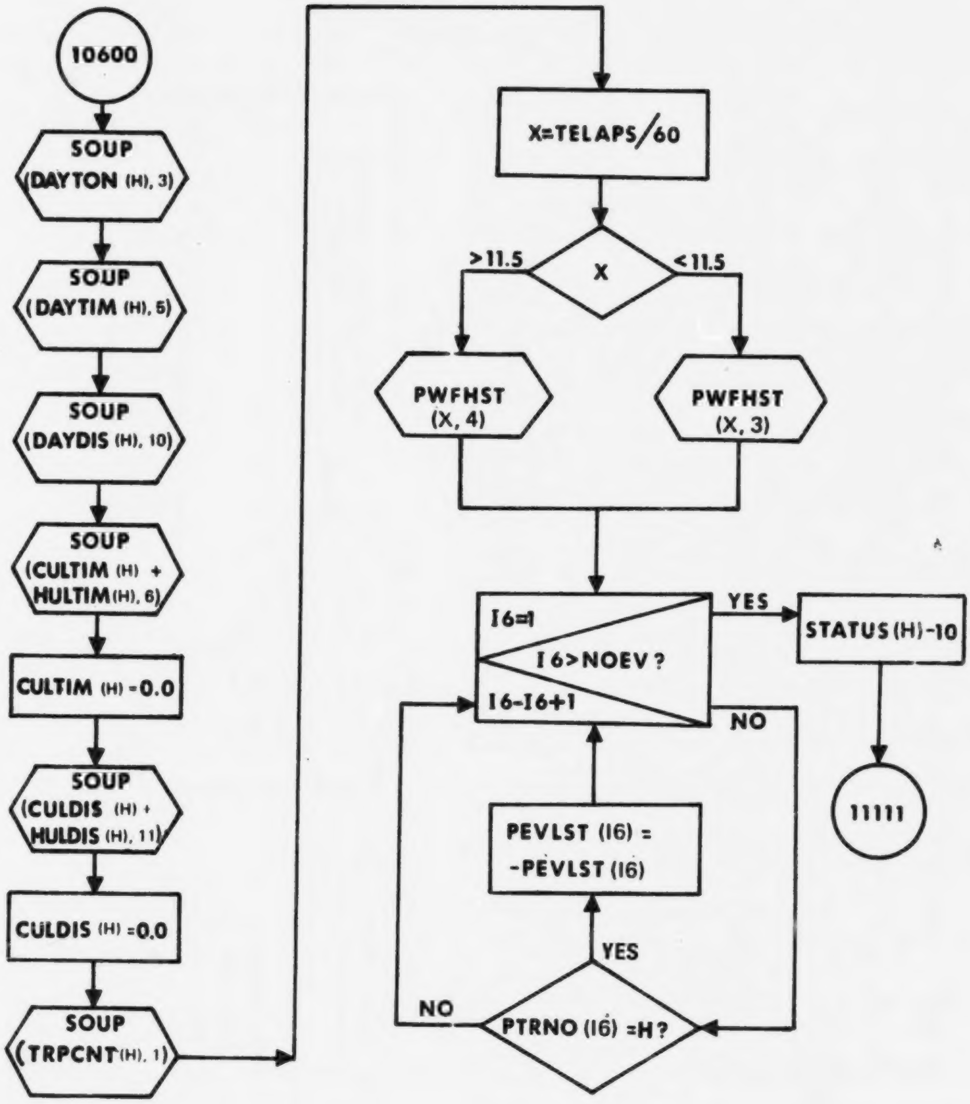

10600
SOUP (DAYTON (H), 3)
SOUP (DAYTIM (H), 5)
SOUP (DAYDIS (H), 10)
SOUP (CULTIM (H) + HULTIM (H), 6)
CULTIM (H) = 0.0
SOUP (CULDIS (H) + HULDIS (H), 11)
CULDIS (H) = 0.0
SOUP (TRPCNT (H), 1)
X=TELAPS/60
X
>11.5
<11.5
PWFHST (X, 4)
PWFHST (X, 3)
I6=1
I6>NOEV?
I6-I6+1
YES
NO
STATUS (H)-10
11111
PEVLST (I6) = -PEVLST (I6)
YES
NO
PTRNO (I6) =H?

Figure A.10 Event Type 6: Truck Arrives at Garage

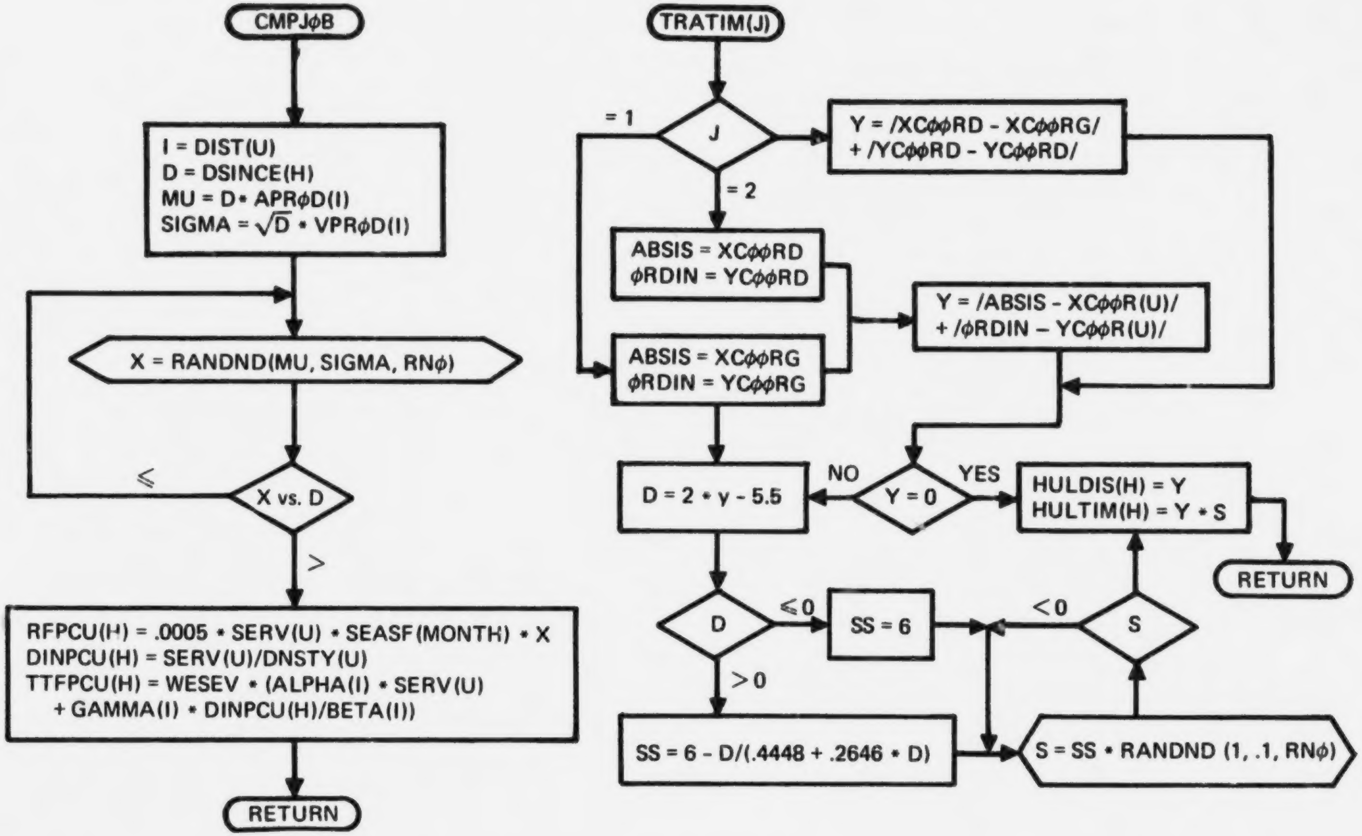

CMPJφB
I = DIST(U)
D = DSINCE(H)
MU = D * APRφD(I)
SIGMA = √D * VPRφD(I)
X = RANDND(MU, SIGMA, RNφ)
X vs. D
≤
>
RFPCU(H) = .0005 * SERV(U) * SEASF(MONTH) * X
DINPCU(H) = SERV(U)/DNSTY(U)
TTFPCU(H) = WESEV * (ALPHA(I) * SERV(U)
+ GAMMA(I) * DINPCU(H)/BETA(I))
RETURN
TRATIM(J)
J
= 1
= 2
Y = /XCφφRD - XCφφRG/
+ /YCφφRD - YCφφRD/
ABSIS = XCφφRD
φRDIN = YCφφRD
ABSIS = XCφφRG
φRDIN = YCφφRG
Y = /ABSIS - XCφφR(U)/
+ /φRDIN – YCφφR(U)/
NO
Y = 0
YES
D = 2 * y - 5.5
HULDIS(H) = Y
HULTIM(H) = Y * S
RETURN
D
≤0
>0
SS = 6
<0
S
SS = 6 - D/(.4448 + .2646 * D)
S = SS * RANDND (1, .1, RNφ)

Figure A.11 CMPJOB: Subroutine to Determine Refuse Amount and Collection Time
TRATIM: Subroutine to Compute Haul Time and Distance

2. Data Generating Subroutines

A. Job Computation (CMPJOB) (Figure A.11)

This subroutine is used to compute the job to be done on a newly entered CU. The job is specified in terms of the three quantities:

RFPCU(H) = Refuse in present CU (in tons)

DINPCU(H) = Distance inside present CU (in miles)

TTFPCU(H) = Time to finish present CU (in minutes) where H = index of the Hth truck.

DINPCU, mileage traveled while collecting, is computed from the quotient services in the CU/service density therein in services per mile.

TTFPCU, is in the present version of this subroutine, computed from a regression formula which is discussed above in section II, 1.D(a).

For further details, see detailed flowchart.

USAGE: CALL CMPJOB: all inputs and outputs are in COMMON and labeled COMMON sections.

B. Travel Time and Distance (TRATIM) (Figure A.11).

This subroutine is used in each event program section where the possibility of truck movement exists. It is also used in the outer program when setting up the initial trip of a truck from the garage to its first CU of the day.

USAGE: CALL TRATIM(J)

where J = 1 denotes garage to CU
J = 2 denotes disposal site to CU or *vice versa*, and
J = 3 denotes disposal site to garage.

All other inputs and outputs are carried in COMMON and labeled COMMON sections.

METHOD: (See also section II, 1.D.(d) of this appendix.)

Let Y = Distance to be traveled. The source cited above gives a speed vs. distance curve based on total haul distance to and from the disposal site. The distance Y is a one-way distance in all three cases (J) above. For J = 1, 3 there is no round trip at all and for case J = 2, it appeared an unwarranted complication to have to reach ahead to find out where the truck might be going next. Therefore, to approximate round trip distance, Y is doubled $m_h = 2 \cdot Y$) in all cases and the computation of speed is based on m_h.

For Distances

1) D = 0, the final results

   HULDIS(H) = 0 (haul distance Hth truck-miles)

   HULTIM(H) = 0 (haul time Hth truck-minutes)

   are returned.

2) $0 < m_h \leq 5.5$, let $V^{-1} = 6$ minutes per mile.

3) $5.5 < m_h$, let $D' = m_h - 5.5$ and let $V^{-1} =$

   $6 - \dfrac{D'}{0.4448 + 0.2646\, D'}$ minutes per mile.

In cases 2) and 3) S is further modified by letting $V^{-1} \cdot N(1, 0.1)$ replace V^{-1}.

Finally, HULDIS(H) = Y(miles) and HULTIM(H) = $Y \cdot V^{-1}$(minutes). Note that distances are computed by the metropolitan metric as described in Section II, 1.D.(c) of this appendix.

For further details, see flowchart.

3. Auxiliary Subroutines

A. Capacity Rule (CAPRUL) (Figure A.12)

This subroutine is used to test the refuse present (either just generated or remaining to be collected) in a given CU against the amount of refuse already collected by the truck and the capacity of the truck. The quantity returned is PROP(H), the proportion of the present CU that the Hth truck can load. The steps of the determination are as follows:

1) Call the FINEXU (Find Next Unit) subroutine to determine whether this is or is not the last unit for today.

2) a) If this is the last unit for today, determine if the sum of the refuse in this collection unit and the refuse already loaded exceed the truck's capacity by more than 5%. If yes, proceed to step 3; if no, set PROP(H) = 1.0.

b) If this is not the last unit for today, proceed to step 3 .

3) Generate a Pseudo-capacity = truck capacity × r where r is a random variable $\sim N(.95, .05)$.

4) Set PROP(H) = ratio of unused pseudo-capacity to refuse present in this collection

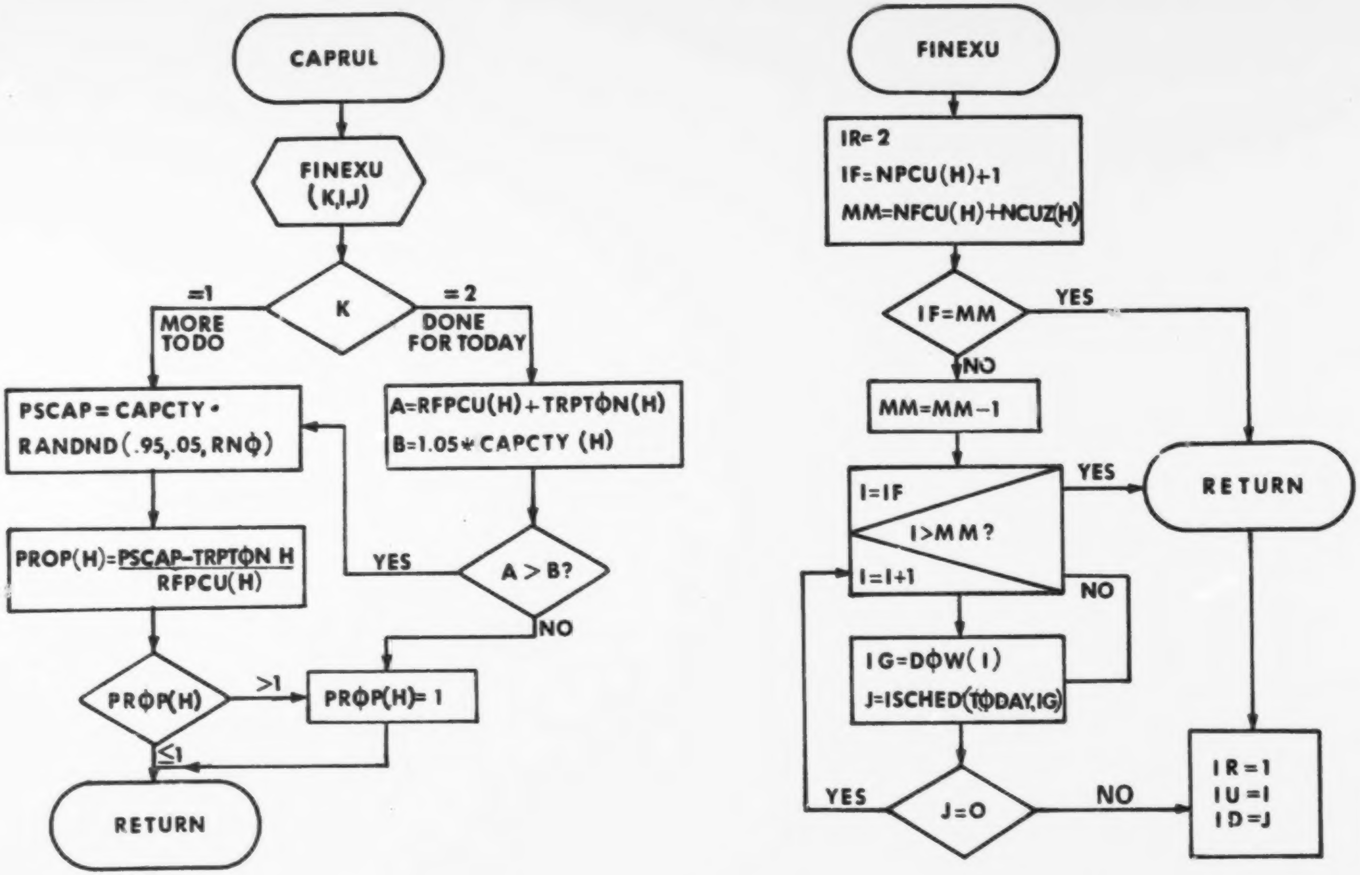

CAPRUL
FINEXU (K,I,J)
K
=1
MORE TODO
=2
DONE FOR TODAY
PSCAP = CAPCTY • RANDND(.95,.05, RNΦ)
A=RFPCU(H)+TRPTΦN(H)
B=1.05 * CAPCTY (H)
PROP(H)=PSCAP-TRPTΦN H / RFPCU(H)
A > B?
YES
NO
PRΦP(H)
>1
≤1
PRΦP(H)= 1
RETURN
FINEXU
IR= 2
IF=NPCU(H)+1
MM=NFCU(H)+NCUZ(H)
IF=MM
YES
NO
MM=MM-1
I=IF
I>MM?
I=I+1
YES
NO
RETURN
IG=DΦW(I)
J=ISCHED(TΦDAY,IG)
J=O
YES
NO
IR=1
IU=1
ID=J

Figure A.12 CAPRUL: Capacity Rule Subroutine
FINEXU: Find Next CU Subroutine

unit, i.e.

$$PROP(H) = \frac{\text{PSEUDO-CAPACITY - REFUSE LOADED}}{\text{REFUSE IN CU}}$$

5) If PROP(H) now exceeds 1.0, reduce it to 1.0.

USAGE: CALL CAPRUL (all inputs and outputs
in COMMON).

B. Find Next Unit (FINEXU) (Figure A.12)

This subroutine, as its name suggests, finds the next unit to be serviced by a given truck on a given day, or it indicates that the truck is now working on its final CU and there is no next unit.

The inputs of this subroutine are:

H = **Truck** number

NPCU(H) = Number of present CU for truck H

NCUZ(H) = Number of collection units in the zone of truck H

DOW(U) = Number of the schedule controlling the servicing of collection unit unit U.

ISCHED(TODAY,DOW(U)) = An array entry either = 0, or > 0

If = 0, this CU(U) is not serviced TODAY.

If > 0, this CU(U) is serviced today and this number is the number of the days since the last collection.

The inputs are sent to the subroutine via unlabeled and labeled COMMON statements.

USAGE: CALL FINEXU (IR, IU, ID)

The items of the calling sequence are the only outputs returned by the subroutine. These are:

IR : = 2 This is the last unit of the day for this truck today. Ignore IU and ID.

= 1 There is a next unit, and

IU : = The number of the next unit found

ID : = The number of days since this unit was last serviced.

C. General Histogram Reader (HISTOG) (Figure A.13)

This subroutine is used only once in the present version of the program. It is available, however, for any future expansion to return a random variable from any arbitrary probability density function. The p.d.f. is presented as a vector of densities representing a histogram. The sum of the entries must equal

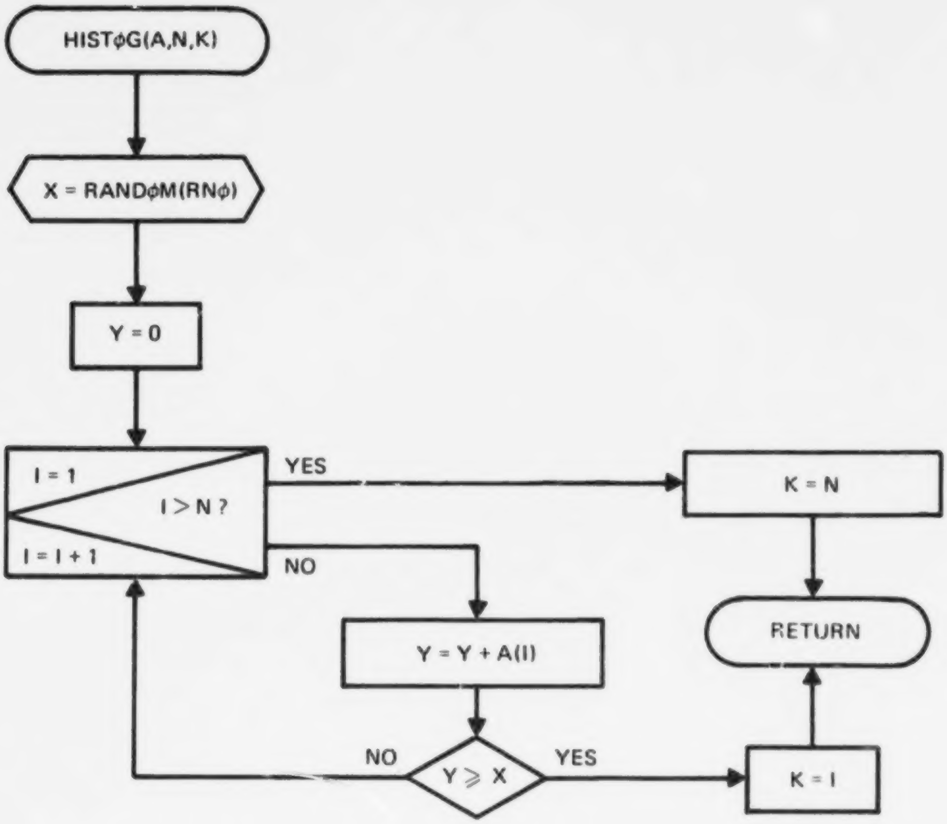

HISTφG(A,N,K)
X = RANDφM(RNφ)
Y = 0
I = 1
I > N ?
I = I + 1
YES
NO
K = N
RETURN
Y = Y + A(I)
NO
Y ⩾ X
YES
K = I

Figure A.13 HISTOG: General Histogram Reader

1.0. The item returned to the calling program is the index K of the histogram which has been chosen arbitrarily. The main program may then use K in a formula to determine the random argument (equally-spaced) or use K to look up the argument from a list of (unequally-spaced) arguments.

USAGE: CALL HISTOG (A, N, K)

A is the vector of densities

N is the length of A

K is the index as described above

METHOD: Generate a random $0 \leq u \leq 1$. Find largest index K such that

$$\sum_{i=1}^{K} A_i \leq u$$

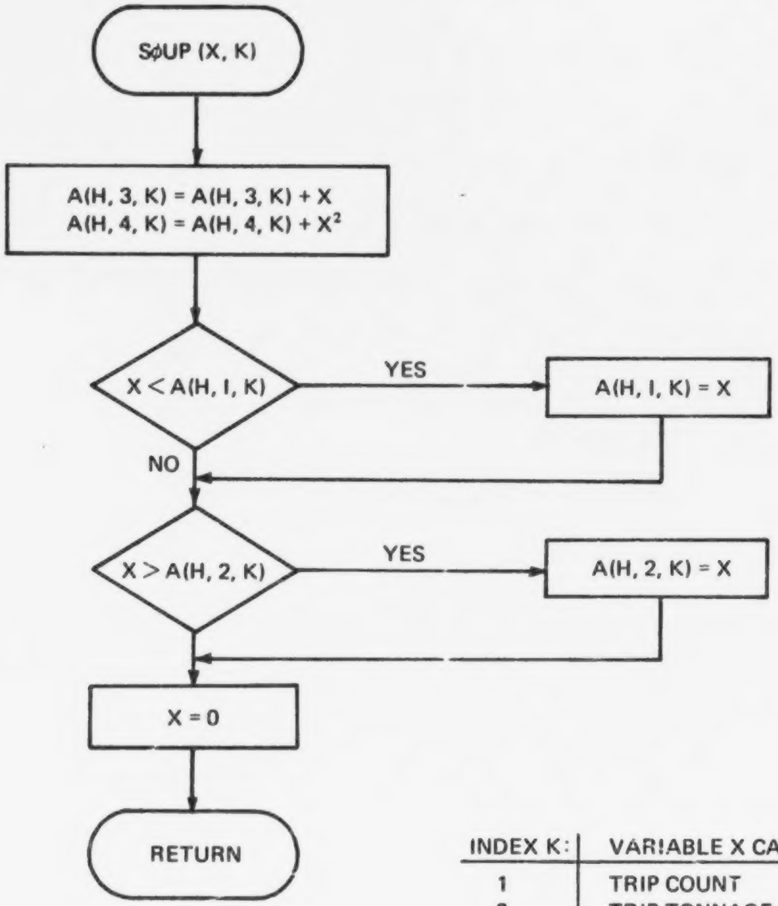
D. General Statistics Accumulator (SOUP)

(Figure A.14)

This subroutine accumulates the minimum, maximum, sum, and sum of squares of (at present) 12 different variables for up to 64 different trucks:

USAGE: CALL SOUP (X, K)

X is variable to be accounted for


SØUP (X, K)
A(H, 3, K) = A(H, 3, K) + X
A(H, 4, K) = A(H, 4, K) + X²
X < A(H, I, K)
YES
A(H, I, K) = X
NO
X > A(H, 2, K)
YES
A(H, 2, K) = X
X = 0
RETURN

LEGEND

INDEX H: TRUCK NUMBER
SECOND INDEX: 1.. MIN. (X)
2.. MAX. (X)
3.. ΣX
4.. ΣX^2

INDEX K:	VARIABLE X CATEGORY
1	TRIP COUNT
2	TRIP TONNAGE
3	DAILY TONNAGE
4	TRIP TIME
5	DAILY TIME
6	HAUL TIME
7	DISPOSAL SITE (DUMP) TIME
8	UNLOADING WAIT TIME
9	TRIP DISTANCE
10	DAILY DISTANCE
11	HAUL DISTANCE
12	HAUL EFFICIENCY

Figure A.14 SOUP: General Statistics Accumulator

K is the variable type ($1 \leq K \leq 12$)

H is the truck number is contained in unlabeled COMMON.

At the end of appropriate time periods, the sums and sums of squares may be used for computing averages and second moments.

The 12 variables represented by the index K are listed in Figure A-14.

E. Report Writer (REPORT)

This subroutine is the largest in the program, and it is neither feasible nor desirable to discuss it here in detail. The end result of its operation may be inferred from a perusal of a sample printout and the description thereof which appear below.

F. Frequency Histogram Packer (PWFHST) (Figure A.15)

This program is used to create frequency histograms of arbitrary performance characteristics.

USAGE: CALL PWFHST (X, I)

where X = a particular value of the performance characteristic

I = the number of the histogram type in which X will be counted

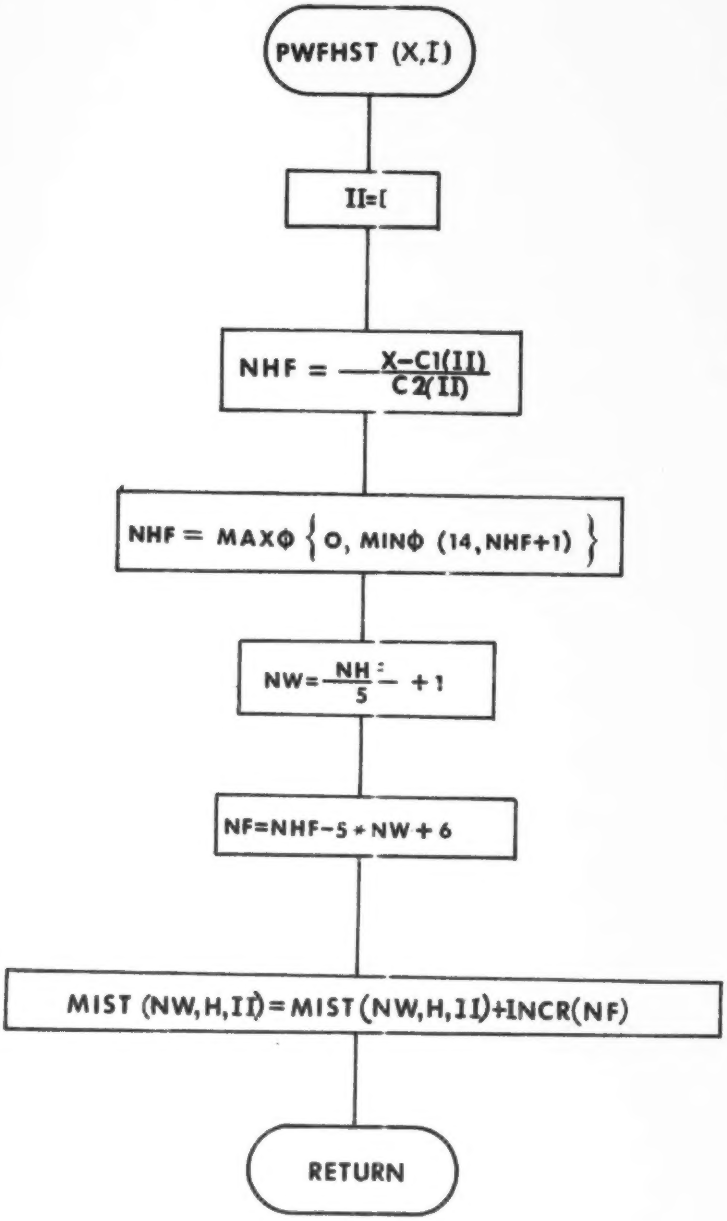

PWFHST (X,I)
II=I
NHF = $\frac{X-C1(II)}{C2(II)}$
NHF = MAXΦ {O, MINΦ (14,NHF+1)}
NW = $\frac{NHF}{5}$ + 1
NF=NHF−5∗NW+6
MIST (NW,H,II)=MIST(NW,H,II)+INCR(NF)
RETURN

Figure A.15 PWFHST: Frequency Histogram Packer

H = truck number (transferred in blank COMMON).

The program provides for up to 18 such histograms (index I above) for each of 64 trucks. Each such histogram consists of 15 "slots" as follows:

Given for each histogram type (I), two constants C_1 and C_2 ...

SLOT #	is used to count x such that
1	$\text{"}-\infty\text{"} < x < C_1$
2	$C_1 \leq x < C_1 + C_2$
.	.
.	.
.	.
14	$C_1 + 12C_2 \leq x < C_1 + 13C_2$
15	$C_1 + 13C_2 \leq x < \text{"}+\infty\text{"}$.

The storage requirements, if each slot occupied a complete computer word for 18 histogram types for 64 trucks would be 17280 (=18 × 64 × 15) words. This has been cut down to 3458 (= 18 × 64 × 3) words by packing slots at five seven-bit fields per word. Thus, this routine will work on any computer having 35 or more binary bits per word exclusive of the sign bit. This routine will not work on a decimal computer.

It is the program user's responsibility to design his runs in such a manner that the count in any given slot does not exceed 127 (= 1 111 111 binary). This can be accomplished in part by splitting the range that is expected for x so that different parts of the range belong to different histogram types. Thus, in the program histogram types 1 and 2 describe the frequency distribution of capacity over the range 0 - 60% and 60 - ∞%, respectively while histogram types 3 and 4 describe the frequency distribution of length of work day over the ranges 0 - 11.5 hours and 11.5 - +∞ hours respectively.

For information on adding new histogram types, see "Block Data Section (BDHST)" below (4.C).

G. Frequency Histogram Unpacker (UNPWFH) (Figure A.16)

This is a companion subroutine to the immediately foregoing subroutine. The function is simply to take the 3 five-slot words pertaining to a certain

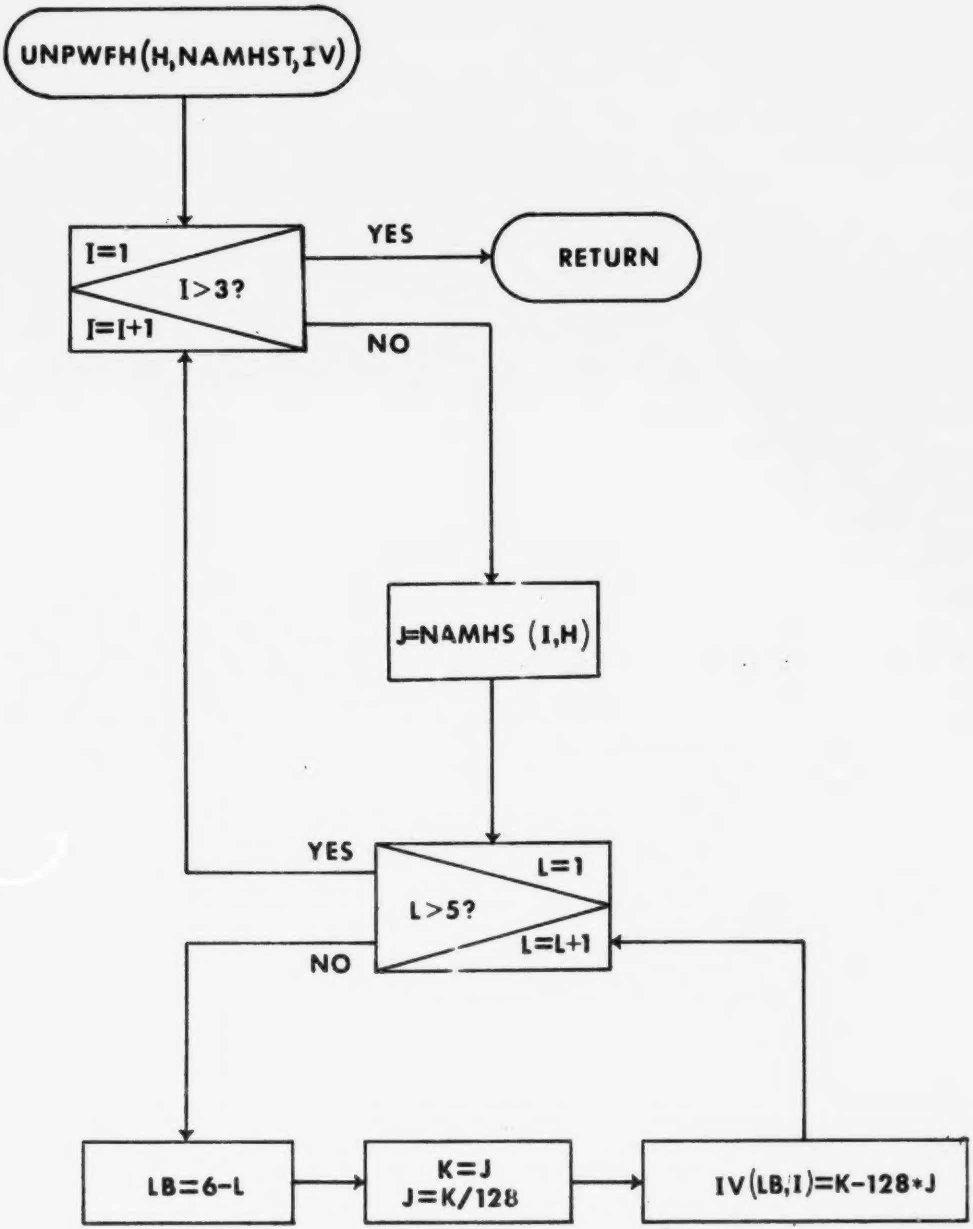

UNPWFH(H,NAMHST,IV)
I=1
I>3?
I=I+1
YES
RETURN
NO
J=NAMHS (I,H)
YES
L=1
L>5?
L=L+1
NO
LB=6-L
K=J
J=K/128
IV(LB,I)=K-128*J

Figure A.16 UNPWFH: Frequency Histogram Unpacker

truck and store the 15 slots in 15 consecutive computer words, so that they may be printed out by REPORT if IHISTO = 0 (option). As above, this routine will not work on a decimal computer.

The histograms are stored in an array called MIST which is dimensioned either 192 × 18 or 3 × 64 × 18 contained in a labeled common section HIST.

USAGE: CALL UNPWFH (H, NAMHST, IV)

H(integer) = truck number

NAMHST = MIST (1, J) for Jth histogram when calling program has MIST dimensioned 192 × 18; = MIST (1, 1, J) when calling program has MIST dimensioned 3 × 64 × 18.

IV = Name of 15 word vector for receiving unpacked histogram "slots".

4. Miscellaneous Subroutines

A. Random Number Generators

a) RANDOM

USAGE: Y = RANDOM (RNO)

RNO is an integer used as the "seed" for generating pseudo-random integers. Y, the value returned, is a floating point (real) variable : $0 < Y < 1$.

This routine is not provided with the program since it is too highly-machine-dependent. It is possible to write a FORTRAN version that will operate, say, on a 35 bit (plus sign) computer having an 8-bit excess-128 exponent and a 27 bit mantissa, but although the program might be FORTRAN, compiled and run on any other type of computer, it would produce meaningless results.

This should prove little hinderence since pseudo-random number generators are available for almost any computer. Thus, if a given computer has a routine called Y = UNIDRV(X) (uniformly distributed random variable), the following program can be quickly written:

```
FUNCTION RANDOM (X)
RANDOM = UNIDRV (X)
RETURN
END
```

b) RANDND

USAGE: Z = RANDND(XMU, SIGMA, RNO)

Returns Z drawn from $N(\mu, \sigma)$ where

μ = XMU

σ = SIGMA

RNO is the seed used to generate two uniformly distributed random variables U1, U2 $\epsilon(0, 1)$ using the function subprogram RANDOM described above.

$$\text{METHOD: } Z = \text{SIGMA} \cdot Y + \text{YMU}$$

$$\text{where } Y \sim N(0, 1)$$

On odd entries generates:

$$X1 = (-2 \log U1)^{1/2} \sin 2\pi U2$$

(Natural log)

$$X2 = (-2 \log U1)^{1/2} \cos 2\pi U2$$

On odd entries sets $Y = X1$ in formula above.
On even entries sets $Y = X2$ left from prior odd entry and uses formula above.

B. Three Decimal Place Rounder (RND3PL)

This routine is used to round a floating point number to 3 decimal places. This can be done in many cases simply by adding 0.0005, or by printing the number with a format "FW.3." On the other hand, if several such numbers are to be added and their sum is to reflect accurately the sum of its rounded components, the decimal places beyond the third must also be removed.

The most accurate way to do this is in machine language. The FORTRAN routine provided here is a compromise and is for positive numbers only.

C. Block Data Section (BDHST)

This is a non-executed program segment that allows constants and alphanumeric labels to be compiled into the labeled common block, /LHST/ used by PWFHST and REPORT.

The items loaded are:

NHIST = The number of histograms currently generated by the program. This is presently set = 4, but may be increased up to 18 .

MXAHST = A 15 × 18 array containing the alphanumeric labeling for the 15 intervals of up to 18 histograms.

LHN = A 3 × 18 array each column of which contains the alphanumeric column heading appearing on the print-out of the arrays. Each column heading is contained in 3 computer words (= 18 characters).

C1 = An 18 place vector whose elements are the upper bounds of the first intervals of each histogram type.

C2 = An 18 place vector whose elements are the common interval lengths for each histogram type.

IV. Preparation of Data for Input

The input data deck makes use of the FORTRAN IV features, NAMELIST, which the user is advised to read.

Runs (i.e., different problems, other variations of a given problem) may be stacked up one behind the other. For all runs after the initial one, only that data need be entered which differs from the data of the immediately preceeding run. Of the sever different data sections to be discussed below, only the first must be encountered on every run.

All data cards must be blank in column 1. Each data section is read under the control of a different NAMELIST the name of which is given on the title line in parentheses, and if read, must be read in the order given here.

1. Run Setup Data (RUNSET)

This data section *may* contain the following items:

a) FOLLOW: This is a 6 position list of indicators which are denoted below as f_i.

$f_i = 0$ means that the i^{th} data section *is not* included in this run.

$\neq 0$ the i^{th} data section is included in this run.

The index i corresponds to the following data sections

i	Data Section (NAMELIST name)
1	WSFDAT
2	TRKDAT
3	DISDAT
4	CUDATA
5	SKEDAT
6	GDSDAT

Example: FOLLOW = 0, 1, 1, 1, 0, 0, ... or FOLLOW = 0, 3*1, 0, 0, ... indicates that WSFDAT (weather and seasonal factor data), SKEDAT (scheduling data), and GDSDAT (garage and disposal site data) will not be read (i.e., will remain unchanged from previous run) for this case.

b) PROBID: This is a case number for identification purposes. Example: PROBID = 10115.

c) NQTRS: These are four integers signifying the number of times each quarter of the year will be simulated.

Example: NQTRS = 2, 3*0, ... indicates that the first quarter (January through March) will be simulated twice and the remaining quarters not at all.

d) IPUNCH = 0 Do not punch results on cards
= 1 Do punch results on cards.

This refers to all results (in a raw form as explained below), including generated histograms.

e) IHISTO = 0 Print histogram results on quarterly report
= 1 Do not print histograms on quarterly report.

f) RNO = seed from which random numbers will be generated.

This is needed only on first case.

<u>Example</u>: RNO = 30517578125 (an integer).

The above inputs are combined as follows (out of order to show that order is unimportant within a section):

```
1 2
  $RUNSET RNO = 30517578125, IHISTC = 1,
 IPUNCH = 0, NQTRS = 2,3*0, PROBID = 10115,
          FOLLOW = 0, 3*1, 0, 0
```

2. <u>Weather and Seasonal Factor Data (WSFDAT)</u>

This data section <u>may</u> contain the following items:

a) SEASF: This is a 12 place list giving the relative refuse production in each of the 12 months vs. a user selected base month. For example, SEASF (1) = 1.0, 0.96, ... 1.09 indicates that February has 96% and December, 109% the refuse production of January.

b) XLIST: This is a 4 place list of relative collection time factors which will be selected randomly according to the weather severity distribution given by the next item. An example will be given below.

c) WEDATA: This is a triply subscripted array, 4 × 3 × 4. The first subscript corresponds to the elements of XLIST, the second to the month of the quarter, and the third to the quarter. The elements are probabilities of occurrence of a slowdown corresponding to XLIST.

Example: Suppose that analysis of the effect of weather on pickup efficiency shows that we may categorize most weather events into four groups with relative work times of 1.00, 1.05, 1.10, 1.15. Then these four entries in the given order are the elements of XLIST. Suppose further that the following data applies to the first three months of the year.

PROBABILITIES OF OCCURRENCE OF
GIVEN PERCENTAGES OF NORMAL WORK RATE

Percent of Normal Pickup Efficiency	Month		
	Jan.	Feb.	Mar.
100	.50	.35	.40
105	.25	.35	.30
110	.15	.25	.20
115	.10	.05	.10
SUM	1.00	1.00	1.00

Then a part of WSFDAT might read

```
1 2
   $WSFDAT XLIST = 1., 1.05, 1.1, 1.15,
WDATA = .5, .25, .15, .1, .35, ..., .2, .1,
```

3. Truck Data (TRKDAT)

This data section *may* contain the following items:

a) NUMTRK: The number of trucks whose operation will be simulated: $0 < \text{NUMTRK} \le 64$.

b) NFCU: Number of first CU in the zone served by each truck. This is a string of up to 64 integers.

c) NCUZ: Number of CU's in the zone served by each truck. This is a string of up to 64 integers.

d) CAPCTY: The capacity of each truck in tons. This is a string of up to 64 numbers: whole numbers will be converted to floating point.

e) CREW: The number of men associated with each truck. This is a string of up to 64 whole numbers.

Example:

```
1 2
  $TRKDAT NUMTRK = 5, NFCU = 1, 5, 9, 13, 17,
      NCUZ = 5*4, CAPCTY = 1, 2, 3., 4., 4,
              CREW = 2, 2, 3, 3, 4
```

4. District Data (DISDAT)

This data section may include the following items:

a) NDIST: Number of districts: $0 < \text{NDIST} \leq 10$.

b) APROD: Average refuse production in pounds per service per day. These are the μ's described in Section II, 1.D.(a) of this appendix. Input as a string of up to 10 numbers.

c) VPROD: Standard deviation of refuse production in like units. These are the σ's of Section II, 1.D.(a). Input as a string of up to 10 numbers.

d) ALPHA
BETA
GAMMA
Each of these is a string of up to 10 numbers. These are the regression constants discussed in Section II, 1.D.(b).

Example:

```
1 2
    $DISDAT NDIST = 2, APROD = 5, 10.,
    VPROD = 1.5, 3.333, ALPHA = 2*10,
       BETA = 2*5, GAMMA = 2*33
```

5. Collection Unit Data (CUDATA)

The number of collection units, NUNITS, *is not* entered as input to the program. The quantity NUNITS is computed as

$$\text{NUNITS} = \sum_{h=1}^{\text{NUMTRK}} \text{NCUZ}_h$$

where NUMTRK and the $\{\text{NCUZ}_h\}$ are entered in section TRKDAT described above. It is the user's responsibility to see that data entered in this section correspond properly with TRKDAT. In particular, it is required that

$$\text{NUMTRK} \leq \text{NUNITS} \leq 1000 \ .$$

The items of data that *may* be entered in this section include the following strings of up to 1000 numbers:

a) DOW: Integers which give the number of the individual schedules (column indices of arrays ISCHED (6, 8). These schedules which are discussed below, dictate which day of the week a CU is to be serviced.

b) DIST: The number of the district within which the CU is wholly contained.

c) SERV: The number of services in each CU.

d) DNSTY: The service density in services per mile.

e) XCOOR
YCOOR: The location of the CU as discussed in Section II, 1.A. of this appendix distances in miles

<u>Example</u>: (In accord with example in TRKDAT above.)

```
1 2
$CUDATA DOW = 1, 1, 2, 2, 1, 1, 2, 2, ..., 1, 1, 2, 2,
DIST = 10*1, 10*2, SERV = 20*500, DNSTY = 10*90, 10*140,
XCOOR = 1, 1.5, 1, .75, ..., 2.3, YCOOR = 1, ..., 5 .
```

6. <u>Scheduling Data (SKEDAT)</u>

This data section <u>may</u> contain the following items:

a) MSCHED: This is a string of seven integers that governs the advancement of the day counter (DAY) and the value of TODAY (= 1 Monday, ..., = 6 Saturday). The following examples illustrate the construction of a

MSCHED string.

<u>Example</u> 1: Suppose that at least one truck works on Tuesdays, Thursdays and Fridays and no trucks work on any other day.

Step 1: Construct a list in ascending order of the "TODAY" values of the days worked. Put a zero at the head of the list and 7 plus the first number after zero at the end.

```
LIST → 0 2 4 5 9
           |   /
           + 7
```

Step 2: Compute first differences (elements):

```
      LIST → 0 2 4 5 9
FIRST DIF. →  2 2 1 4
```

Step 3: Above list write between each pair the sum 1 + the left member of the pair. These are the list indices of the elements:

```
  POSITION →  1 3 5 6
      LIST → 0 2 4 5 9
FIRST DIF. →  2 2 1 4
```

Thus the resultant MSCHED = 2, 0, 2, 0, 1, 4, 0. The positions not computed as above, are filled with zeros. The program does not use these elements.

Example 2: Trucks service some of their CU's on Mondays and Thursdays and the rest on Tuesdays and Fridays.

Step 1: 0 1 2 4 5 8
+ 7

Step 2: 0 1 2 4 5 8
1 1 2 1 3

Step 3: 1 2 3 5 6
0 1 2 4 5 8
1 1 2 1 3

MSCHED = 1, 1, 2, 0, 1, 3, 0 .

b) ISCHED: This is an array 6 × 8 each column of which may constitute a schedule of collection for a CU. Thus in Example 2 above, Monday-Thursday CU's have a schedule 4 0 0 3 0 0 . A zero indicates that the CU is not serviced on the corresponding day of the week. A non-zero conveys the opposite and the value given i.e.

4 and 3 in the example, are the number of days since the last collection.

Example: (Corresponds to Example 2 above).

```
$SKEDAT MSCHED = 1, 1, 2, 0, 1, 3, 0,
   ISCHED = 4, 0, 0, 3, 0, 0,
  ISCHED (1, 2) = 0, 4, 0, 0, 3, 0
```

Referring back to section CUDATA, CU's with a Monday-Thursday collection should have DOW = 1 and those with a Tuesday-Friday collection should have DOW = 2 , for this example.

7. Garage and Disposal Site Data (GDSDAT)

The following items may be entered in this data section:

a) NLD: The number of loading docks, i.e. positions at which a truck may dump its load at the disposal site.

b) XCOORD
YCOORD The location of the disposal site with respect to same origin used for the location of CU's. Distance in miles.

c) XCOORG
YCOORG The location of the garage. See remarks in b) immediately above.

```
$RUNSET PROBID=100001,FOLLOW=1,2,3,4,5,6,NQTRS=1,1,0,0,IPUNCH=1,IHISTO=0,
        RNO=17043598603
$END
$WSFDAT SEASF=12*1.,      XLIST=4*1.,
        WEDATA=1.,3*0.,1.,3*0.,1.,3*0.,1.,3*0.,1.,3*0.,1.,3*0.,
               1.,3*0.,1.,3*0.,1.,3*0.,1.,3*0.,1.,3*0.,1.,3*0.
$END
$TRKDAT NUMTRK=3,NFCU=1,3,5,NCUZ=3*2,CAPCTY=3*3.,CREW=3*2.
$END
$DISDAT NDIST=9,APROD=3*5.,3*10.,3*15.,VPROD=.5,1.5,3.,1.,3.,6.,1.5,4.5,9.,
        ALPHA=9*.7,BETA=9*5.,GAMMA=9*33.
$END
$CUDATA DOW=1,2,1,2,1,2,DIST=1,1,2,2,3,3,SERV=6*350.,DNSTY=6*70.,XCOOR=6*0.,
        YCOOP=6*0.
$END
$SKEDAT MSCHED=1,1,2,0,1,3,0,ISCHED=4,0,0,3,0,0,0,4,0,0,3,0
$END
$GDSDAT NLD=50,XCOORG=0.,YCOORG=0.,XCOORD=0.,YCOORD=0.
$END
```

Table A.1 Example of Input Card Deck

V. Description of Output

1. Printed Output

One or two pages of output are printed for each truck at the end of each simulated quarter. A sample of this printed output is given in Table A.2a and b.

The first page of the output and all items thereon are printed, i.e.,they are not subject to supression via an option entered on the RUNSET card of the input (see above). This option is the input variable IHISTO. The second page reports certain frequency histograms described below and is suppressed by setting IHISTO $\neq$ 0 and permitted if IHISTO = 0.

A. First Page Format

The first line reports truck number, truck capacity, crew size, the number of the quarter (1, 2, 3, or 4) and the 6-place integer problem identification (PROBID).

The next section, entitled "The Job Undertaken by This Truck," gives a brief recapitulation of the input data related to this truck and is readily understood in terms of the description of the input given above.

The third section of the first printed page is

```
REFUSE COLLECTION SIMULATION REPORT FOR TRUCK   1, CAPACITY = 3.00 TONS, CREW = 2 MEN, QTR 1, PROB 100001


T H E   J O B   U N D E R T A K E N   B Y   T H I S   T R U C K

COL.UNIT SVCS DENSTY XCOORD YCOORD AVPROD SDPROD  ALPHA     BETA    GAMMA   DAYS SRVCED


    1   350  70.00   4.00    .00   5.00    .50    .7000   5.0000  33.0000 M     T
    2   350  70.00   4.00    .00   5.00    .50    .7000   5.0000  33.0000   T     F
. . . . . . . . D I G E S T   O F   M A I N   R E S U L T S . . . . . . .


DAYS WORKED   52    PICKUP TIME   14455.998  83.354  AGGR COLRATE 214.348
NUMBER TRIPS  78    HAUL   TIME    2382.471  13.737  PKUP COLRATE 178.667
MIN TRPS/DAY   1    DUMP   TIME     504.523   2.909  AVG  SPEED    15.715
MAX TRPS/DAY   2    WAIT   TIME        .000    .000  EFF CAPACITY   2.075
AVG TRPS/DAY   1.50   TOTAL  TIME  17342.992 100.000  PCT.CAP.UTIL  69.154
                    AVG HRS/DAY       5.559          AVG H-EFFCY    3.282
. . .T A B L E   O F   D E T A I L E D   R E S U L T S. . .

    I T E M          MINIMUM   MAXIMUM   AVERAGE   STD.DE..


TONS CARRIED/TRIP       .268     3.243     2.075     1.015
TONS CARRIED/DAY       2.363     3.935     3.112      .495
PICKUP TIME /TRIP     22.413   278.000   185.333    97.876
PICKUP TIME /DAY     278.000   278.000   278.000      .088
HAUL   TIME /DAY      25.105    66.957    45.817    15.894
DUMP   TIME /TRIP      5.028     8.657     6.469      .993
WAIT   TIME /TRIP       .000      .000      .000      .000
PICKUP MILES/TRIP       .403     5.000     3.333     1.760
PICKUP MILES/DAY       5.000     5.000     5.000      .001
HAUL   MILES/DAY       8.000    16.000    12.000     4.000
MAN*MIN/TON*MILE       1.047    15.753     3.282     3.433
```

Table A.2a Sample of Printed Output, First Page

```
TRUCK   1 CONT'D., FREQUENCY HISTOGRAMS.   RANGES ARE LEFT-CLOSED AND RIGHT-OPEN.   I = +INFINITY.

CAPACITY UTILIZED    CAPACITY UTILIZED    LENGTH OF WORKDAY    LENGTH OF WORKDAY
RANGE   FREQUENCY    RANGE   FREQUENCY    RANGE    FREQUENCY   RANGE    FREQUENCY
  0- 4          0     60-64          0     0.0-            0   11.5-            0
  4- 8          0     64-68          0     4.5-            0   12.0-            0
  8-12          5     68-72          0     5.0-           26   12.5-            0
 12-16          1     72-76          0     5.5-           26   13.0-            0
 16-20          3     76-80          2     6.0-            0   13.5-            0
 20-24          0     80-84          5     6.5-            0   14.0-            0
 24-28          5     84-88          3     7.0-            0   14.5-            0
 28-32          3     88-92         12     7.5-            0   15.0-            0
 32-36          1     92-96         14     8.0-            0   15.5-            0
 36-40          2     96-            9     8.5-            0   16.0-            0
 40-44          0    100-            5     9.0-            0   16.5-            0
 44-48          0    104-            1     9.5-            0   17.0-            0
 48-52          0    108-            1    10.0-            0   17.5-            0
 52-56          0    112-            0    10.5-            0   18.0-            0
 56-60          0    116-I           0    11.0-            0   18.5-I           0
```

Table A.2b Sample of Printed Output, Second Page

called "Digest of Main Results." This consists of three columns of titles and four columns of figures. The meaning of these is as follows:

a) First column of titles:

Days worked -- this may be different for different trucks and should be checked against the scheduling input data. Always a multiple of 13.

Number of trips -- the total number of loads carried to the disposal site in the number of days worked. Travel to and from the garage is not included in this count.

Minimum, maximum and average trips per day -- self explanatory.

b) Second column of titles: Two figures accompany all but the last title in this column. Of the paired columns of figures, the figures on the left are total time in minutes over the quarter, and the figures on the right are percentages of total time spent by this truck in the following categories of time utilization.

1) Pickup Time - total time spent in actual refuse pickup.

2) Haul Time - total time spent in travelling to and from the disposal site and the garage.

3) Dump Time - total time spent at the disposal site discharging loads, exclusive of any time spent waiting for an unloading dock to be vacated.

4) Wait Time - total time spent at the disposal site waiting for an unloading dock to be vacated.

5) Total Time - The sum of 1) through 4).

The final (unpaired) figure in this column gives the average hours worked per day and is computed by dividing 5) above by 60 × days worked.

c) Third column of titles:

AGGR COLRATE - The average value of the aggregate collection rate in man-min per ton. This is CREW × Pickup Time (b.5 above)/total tons collected.

PKUP COLRATE - The average value of the collection rate in man-min per ton during collection only. This is CREW × Pickup Time (b.1 above)/total tons collected.

AVG SPEED - The average speed of this truck in miles per hour during all travelling opera-

EFF CAPACITY -- The effective capacity, i.e. average tons per load.

PCT. CAP. UTIL -- Average percent capacity utilization = 100 × (effective capacity / truck capacity).

AVG H-EFFCY -- The average haul efficiency in units of man·min per ton mile.

The fourth section of the first page is entitled "Table of Detailed Results" and gives the minimum, maximum, average, and standard deviation of the items listed to the left. All times are in minutes per day or per trip.

B. Second Page Format

The second page of printed output is under control of the input option IHISTO in input section RUNSET (= 0 allows printing; ≠ 0 suppresses printing).

The contents of this page are two frequency histograms of two columns each. The first of these histograms gives the trip distribution of percent capacity utilization. The range of percentages covered is from 0 to 116% in 4% intervals (closed on the left and open on the right). The final interval covers the range 116% to + infinity. The sum of the frequencies equals the total number of trips made by this truck.

The second histogram is the day distribution of daily hours worked (length of workday). The range of time is covered as follows: An initial interval going from 0 to 4.5 hours, 28 intervals going from 4.5 to 18.5 hours in half hour increments, and a final interval going from 18.5 hours to + infinity. All intervals are closed on the left and open on the right. The sum of the frequencies equals the number of days worked by this truck.

Many of the upper range limits are omitted due to space restrictions. When such a limit is missing, it is to be interpreted as being equal to the lower limit of the next interval.

The program may be altered to allow up to 14 more such histogram columns to be printed on this page.

2. Punched Card Output

Three main groups of cards are punched and (see sample in Table A.3a and b). The last of these is miscellaneous and will be disposed of forthwith. This group is a single card and takes the form RNO = I I ... I $. This allows re-entry of the random number seed. Its

```
100001  1  1  1   .10000000+01    .20000000+01    .78000000+02    .13000000+03
100001  1  1  2   .26848837+00    .32424309+01    .16182076+03    .41613049+03
100001  1  1  3   .23633165+01    .39347580+01    .16182077+03    .51630927+03
100001  1  1  4   .22412954+02    .27800000+03    .14455998+05    .34264242+07
100001  1  1  5   .27800000+03    .27800000+03    .14455999+05    .40187676+07
100001  1  1  6   .25105352+02    .66956627+02    .23824710+04    .12229296+06
100001  1  1  7   .50280206+01    .86559108+01    .50452307+03    .33402269+04
100001  1  1  8   .99999999+08    .00000000       .00000000       .00000000
100001  1  1  9   .40311068+00    .50000000+01    .25999997+03    .11083873+04
100001  1  1 10   .49999999+01    .50000000+01    .25999998+03    .12999999+04
100001  1  1 11   .80000000+01    .16000000+02    .62400000+03    .83200000+04
100001  1  1 12   .10472496+01    .15753254+02    .25601667+03    .17596428+04
100001  1  1  1   0   0   5   1   3   6   5   3   1   2   0   0   0   0   0
100001  1  1  2   0   0   0   0   2   5   3  12  14   9   5   1   1   0   0
100001  1  1  3   0   0  26  26   0   1   0   0   0   0   0   0   0   0   0
100001  1  1  4   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
100001  1  2  1   .10000000+01    .20000000+01    .78000000+02    .13000000+03
100001  1  2  2   .28031068+00    .31272619+01    .16516003+03    .42529420+03
100001  1  2  3   .16401255+01    .46324477+01    .16516005+03    .54544423+03
100001  1  2  4   .23966926+02    .27800000+03    .14455998+05    .33802528+07
100001  1  2  5   .27800000+03    .27800000+03    .14455999+05    .40187678+07
100001  1  2  6   .25211370+02    .66400317+02    .23701072+04    .12023905+06
100001  1  2  7   .50102654+01    .89504738+01    .50896993+03    .33984541+04
100001  1  2  8   .99999999+08    .00000000       .00000000       .00000000
100001  1  2  9   .43105982+00    .50000000+01    .25999997+03    .10934517+04
100001  1  2 10   .49999999+01    .50000000+01    .25999998+03    .12999999+04
100001  1  2 11   .80000000+01    .16000000+02    .62400000+03    .83200000+04
100001  1  2 12   .10784511+01    .15145876+02    .23043644+03    .13030623+04
100001  1  2  1   0   0   3   3   7   2   2   2   0   1   1   1   1   2   0
100001  1  2  2   0   2   2   0   2   3   6   6  11   9  10   2   0   0   0
100001  1  2  3   0   0  26  26   0   0   0   0   0   0   0   0   0   0   0
100001  1  2  4   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
100001  1  3  1   .10000000+01    .20000000+01    .83000000+02    .14500000+03
100001  1  3  2   .23925214+00    .32398284+01    .17761865+03    .43719959+03
100001  1  3  3   .10711442+01    .57618904+01    .17761866+03    .68819489+03
100001  1  3  4   .20779839+02    .27800000+03    .14455998+05    .30503527+07
100001  1  3  5   .27800000+03    .27800000+03    .14455998+05    .40187676+07
100001  1  3  6   .25944597+02    .69037294+02    .25311623+04    .13536423+06
100001  1  3  7   .50032482+01    .89446030+01    .52190964+03    .33728160+04
100001  1  3  8   .99999999+08    .00000000       .00000000       .00000000
100001  1  3  9   .37373811+00    .50000000+01    .25999996+03    .98673485+03
100001  1  3 10   .49999999+01    .50000000+01    .25999999+03    .12999999+04
100001  1  3 11   .80000000+01    .16000000+02    .66400000+03    .92800000+04
100001  1  3 12   .93660405+00    .14724058+02    .19918403+03    .78947151+03
100001  1  3  1   0   1   0   0   1   3   2   4   3   4   1   4   1   2   1
100001  1  3  2   0   5   2   3   5   2   2   6  11  12   7   1   0   0   0
100001  1  3  3   0   0  21  30   1   0   0   0   0   0   0   0   0   0   0
100001  1  3  4   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
 RNO=-14790672812
```

Table A.3a Listing of Punched Card Output

```
100001  2  1  5    .27800000+03      .27800000+03      .14455999+05      .40187678+07
100001  2  1  6    .26378957+02      .68114225+02      .23435470+04      .11761772+06
100001  2  1  7    .50089035+01      .89808693+01      .49058195+03      .32456787+04
100001  2  1  8    .99999999+08      .00000000         .00000000         .00000000
100001  2  1  9    .48503287+00      .50000000+01      .25999997+03      .11191501+04
100001  2  1 10    .49999999+01      .50000000+01      .25999999+03      .12999999+04
100001  2  1 11    .80000000+01      .16000000+02      .60800000+03      .79360000+04
100001  2  1 12    .10723050+01      .13217054+02      .22642089+03      .11969740+04
100001  2  1  1    0    0    1    5    3    4    5    4    1    1    0    0    0    0    0
100001  2  1  2    0    0    0    0    2    3    3   13   15   10    5    1    0    0    0
100001  2  1  3    0    0   28   24    0    0    0    0    0    0    0    0    0    0    0
100001  2  1  4    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
100001  2  2  1    .10000000+01      .20000000+01      .73000000+02      .11500000+03
100001  2  2  2    .21068219+00      .31613381+01      .16485691+03      .42945427+03
100001  2  2  3    .19861793+01      .47026019+01      .16485692+03      .54573732+03
100001  2  2  4    .18218786+02      .27800000+03      .14455998+05      .34463365+07
100001  2  2  5    .27800000+03      .27800000+03      .14455999+05      .40187677+07
100001  2  2  6    .23551430+02      .67298446+02      .22507513+04      .10987305+06
100001  2  2  7    .50114861+01      .88226891+01      .46982552+03      .31091798+04
100001  2  2  8    .99999999+08      .00000000         .00000000         .00000000
100001  2  2  9    .32767601+00      .50000000+01      .25999997+03      .11148286+04
100001  2  2 10    .49999999+01      .50000000+01      .25999999+03      .12999999+04
100001  2  2 11    .80000000+01      .16000000+02      .58400000+03      .73600000+04
100001  2  2 12    .97839272+00      .19064691+02      .18579383+03      .10794670+04
100001  2  2  1    0    2    0    0    2    2    2    5    1    0    2    2    1    1    0
100001  2  2  2    0    3    1    1    0    2    4   11    9   10    9    3    0    0    0
100001  2  2  3    0    0   31   21    0    0    0    0    0    0    0    0    0    0    0
100001  2  2  4    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
100001  2  3  1    .10000000+01      .30000000+01      .87000000+02      .16300000+03
100001  2  3  2    .38243016+00      .31084740+01      .18677709+03      .46672246+03
100001  2  3  3    .98804667+00      .67102498+01      .18677711+03      .76323769+03
100001  2  3  4    .23371887+02      .27800000+03      .14455998+05      .29921704+07
100001  2  3  5    .27800000+03      .27800000+03      .14455999+05      .40187678+07
100001  2  3  6    .26300047+02      .90999763+02      .26550788+04      .15235651+06
100001  2  3  7    .50061951+01      .89872952+01      .54202033+03      .34543004+04
100001  2  3  8    .99999999+08      .00000000         .00000000         .00000000
100001  2  3  9    .42035769+00      .50000000+01      .25999996+03      .96791393+03
100001  2  3 10    .49999999+01      .50000000+01      .25999999+03      .13000000+04
100001  2  3 11    .80000000+01      .24000000+02      .69600000+03      .10432000+05
100001  2  3 12    .89723042+00      .10101058+02      .21278910+03      .85498061+03
100001  2  3  1    0    0    0    3    2    3    4    3    3    3    0    1    2    2    3
100001  2  3  2    0    2    4    3    0    2    5    7   17   11    7    0    0    0    0
100001  2  3  3    0    0   20   27    5    0    0    0    0    0    0    0    0    0    0
100001  2  3  4    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
 RNO= 21752781779
```

Table A.3b Listing of Punched Card Output (Cont.)

output is not under the control of an option, and one such card is punched for each quarter simulated.

A. Detailed Data Cards

The first group of cards consists of twelve cards per truck per quarter. The format of these twelve cards is as follows:

CARD COLUMNS								
	1-6	7-9	10-12	13-15	16-30	31-45	46-60	61-75
	PROBID	QTR	TRKNO	CARDNO	ITMIN	ITMAX	ITSUM	ITSSQ
FORMAT	I6	I3	I3	I3	E15.8	E15.8	E15.8	E15.8

PROBID - same as on input section RUNSET (q.v.).

QTR - the number of the quarter of the year.

TRKNO - the number of the truck = 1, 2, ..., NUMTRK.

CARDNO - the card number in this series of 12 = 1, 2, ..., 12.

ITMIN - the minimum value of the item.

ITMAX - the maximum value of the item.

ITSUM - the sum of the values of the item (not the average).

ITSSQ - the sum of the squares of the values of the item.

Before enumeration of the items, note that when finding a minimum one sets the location where it is to be stored to a very large number (in this case, 99,999,999) and replaces it each time a smaller value is found. If no positive value of the item is ever generated, we punch out ITMIN = 99,999,999 and ITMAX through ITSSQ = 0. The items are as follows:

Card #	
1	The trip count.
2	Tonnage on a trip basis.
3	Tonnage on a daily basis.
4	Pickup time on a trip basis.
5	Pickup time on a daily basis.
6	Haul time on a daily basis.
7	Dump time on a trip basis.
8	Wait time on a trip basis.
9	Pickup mileage on a trip basis.
10	Pickup mileage on a daily basis.
11	Haul mileage on a daily basis.
12	Haul efficiency (man-min per ton-mile) on a round trip basis.

B. Frequency Histogram Cards

The second group consists of four cards per truck per quarter. These contain the frequencies of fifteen intervals per card.

CARD COLUMNS					
	1-6	7-9	10-12	13-15	16 - 75
	PROBID	QTR	TRKNO	CARDNO	FIFTEEN CONSECUTIVE FREQUENCIES
FORMAT	I6	I3	I3	I3	15 or 15

PROBID, QTR, TRKNO are as above.

CARDNO - the card number in this series of 4 = 1, 2, 3, 4 .

FREQUENCIES - are as described above in the second page of printed output. Cards 1 and 2 are the capacity utilization histogram and cards 3 and 4 are the length of workday histogram.

DETAILED LISTING OF SIMULATION PROGRAM

```
C DECLARATIVES AND SPACE MANAGEMENT
C
C 1.COMMON STATEMENTS
C
      COMMON RNO,H,U,TODAY
      COMMON /TSR/NOEV,TELAPS,PEVLST(500),TIMLST(500),PTRNO(500)
      COMMON /LOCS/XCOORD,YCOORD,XCOORG,YCOORG,XCOOR(1000),YCOOR(1000)
      COMMON /CPRL/TRPTON(64),RFPCU(64),PROP(64),CAPCTY(64),TTFPCU(64)
      COMMON /CUNJB/ DIST(1000),SERV(1000),DNSTY(1000),
     A    APROD( 10),VPROD( 10),ALPHA( 10),BETA( 10),GAMMA( 10),
     B    DSINCE( 64),CREW( 64),DINPCU( 64),
     C    SEASF(12),MONTH,WESEV
      COMMON /HAUL/HULTIM(64),HULDIS(64)
      COMMON /RPRT/NUMTRK,QTR,PROBID,IF1P,IF0H
      COMMON /SCHED/DOW(1000),NFCU(64),NCUZ(64),NPCU(64),ISCHED(6,8),
     A             MSCHED(7)
      COMMON /HIST/MIST(192,18)
      COMMON /O/
     A          TCTMIN( 64),TCTMAX( 64),TCTSUM( 64),TCTSSQ( 64),
     B           TTMIN( 64), TTMAX( 64), TTSUM( 64), TTSSQ( 64),
     C           DTMIN( 64), DTMAX( 64), DTSUM( 64), DTSSQ( 64),
     F          TTIMIN( 64),TTIMAX( 64),TTISUM( 64),TTISSQ( 64),
     G          DTIMIN( 64),DTIMAX( 64),DTISUM( 64),DTISSQ( 64),
     H           CTMIN( 64), CTMAX( 64), CTSUM( 64), CTSSQ( 64),
     I          DMPMIN( 64),DMPMAX( 64),DMPSUM( 64),DMPSSQ( 64),
     J           UDMIN( 64), UDMAX( 64), UDSUM( 64), UDSSQ( 64),
     L           TDMIN( 64), TDMAX( 64), TDSUM( 64), TDSSQ( 64),
     M           DDMIN( 64), DDMAX( 64), DDSUM( 64), DDSSQ( 64),
     N           CDMIN( 64), CDMAX( 64), CDSUM( 64), CDSSQ( 64),
     O          HEFMIN( 64),HEFMAX( 64),HEFSUM( 64),HEFSSQ( 64)
      COMMON/Z/ TRPCNT( 64),TRPTIM( 64),TRPDIS( 64),
     A          DAYTON( 64),DAYTIM( 64),DAYDIS( 64),
     B            DUMP( 64),CULTIM( 64),CULDIS( 64), UDBNT( 64)
C
```

```
C 2.TYPE DECLARATIVES
C
      INTEGER H,U,EVENT,DOW,PEVLST,PTRNO,TODAY
      INTEGER EOM,QTR,DAY
      INTEGER DIST,DSINCE,RNO
      INTEGER STATUS
      INTEGER FOLLOW(6)
      INTEGER PROBID
      LOGICAL SKPDK
C
C 3.ARRAYS NOT APPEARING IN COMMON STATEMENTS
C
      DIMENSION XLIST(4),LDBI(64),STATUS(64),WEDATA(4,3,4)
      DIMENSION NQ(4)
      DIMENSION NQTRS(4)
      DIMENSION A(64,48),B(64,10)
      EQUIVALENCE (A,TCTMIN)
      EQUIVALENCE (B,TRPCNT)
      EQUIVALENCE (IPUNCH,IF1P),(IHISTO,IF0H)
C
C 4.NAMELIST STATEMENTS
C
      NAMELIST/RUNSET/FOLLOW,PROBID,NQTRS,IPUNCH,IHISTO,RNO
      NAMELIST/WSFDAT/SEASF,XLIST,WEDATA
      NAMELIST/TRKDAT/NUMTRK,NFCU,NCUZ,CAPCTY,CREW
      NAMELIST/DISDAT/NDIST,APROD,VPROD,ALPHA,BETA,GAMMA
      NAMELIST/CUDATA/DOW,DIST,SERV,DNSTY,XCOOR,YCOOR
      NAMELIST/SKEDAT/MSCHED,ISCHED
      NAMELIST/GDSDAT/NLD,XCOORG,YCOORG,XCOORD,YCOORD
C
```

```
C BEGIN DATA READ-IN ROUTINES
C
  111 READ(5,RUNSET)
      DO 119 J=1,4
  119 NQ(J) = NQTRS(J)
      DO 100 I=1,6
      IF(FOLLOW(I).EQ.0)GO TO 100
      GO TO(101,102,103,104,105,106),I
  101 READ(5,WSFDAT)
      GO TO 100
  102 READ(5,TRKDAT)
      IF(NUMTRK.LE.0.OR.NUMTRK.GT.64)GO TO 111
      NUNITS = 0
      DO 1021 J=1,NUMTRK
 1021 NUNITS = NUNITS + NCUZ(J)
      IF(NUNITS.LT.NUMTRK.OR.NUNITS.GT.1000)GO TO 111
      GO TO 100
  103 READ(5,DISDAT)
      IF(NDIST.LE.0.OR.NDIST.GT.10)GO TO 111
      GO TO 100
  104 READ(5,CUDATA)
      GO TO 100
  105 READ(5,SKEDAT)
      GO TO 100
  106 READ(5,GDSDAT)
  100 CONTINUE
C
```

```
C END DATA READ-IN ROUTINES--BEGIN MAJOR LOOP OF RUN
C
  999 QTR=1
      SKPDK=NLD.GE.NUMTRK
  300 IF(NQ(QTR).NE.0)GO TO 302
      QTR=QTR+1
      IF(QTR.NE.5)GO TO 300
C
C RETURN FOR DATA FOR NEXT RUN
C
      GO TO 111
C
  302 NQ(QTR)=NQ(QTR)-1
      MONTH=3*QTR - 2
      TODAY=0
      DAY=0
      MOQ=1
      EOM=32
C
C CLEAR DATA ACCUMULATORS
C
      DO 1 I=1,64
      DO 3 K=1,10
    3 B(I,K)=0.0
      DO 1 J=1,45,4
      A(I,J)=99999999.0
      A(I,J+1)=0.0
      A(I,J+2)=0.0
    1 A(I,J+3)=0.0
      NOEV=0
      DO 2 I=1,64
    2 LDBI(I)=0
      DO 444 J=1,18
      DO 444 I=1,192
  444 MIST(I,J)=0
```

```
C ENTER HERE AT END OF DAY
22222 ISW=MSCHED(TODAY+1)
      DAY=DAY + ISW
      IF(DAY.GE.EOM)GO TO 22001
22000 TODAY=MOD(TODAY+ISW,7)
C BEGINNING OF DAY--GET TODAY'S WEATHER SEVERITY
  200 CALL HISTOG(WEDATA(1,MOQ,QTR),4,IR)
      WESEV=XLIST(IR)
      TELAPS=0.0
C
C FETCH FIRST TASK FOR EACH TRUCK
C
      DO 201 H=1,NUMTRK
      TRPTON(H)=0.0
      NPCU(H)=NFCU(H)-1
      CALL FINEXU(K,U,J)
      GO TO(203,202),K
  202 STATUS(H)=1
      GO TO 201
  203 NPCU(H)=U
      DSINCE(H)=J
      CALL TRATIM(1)
      CALL PUT(HULTIM(H),2)
      STATUS(H)=2
  201 CONTINUE
C
```

```
C BEGIN MAIN ITERATIVE LOOP OF DAILY SIMULATION-GET EVENT FROM TSR
C
11111 IF(NOEV.EQ.0)GO TO 22222
      T=TIMLST(1)
      EVENT=PEVLST(1)
      H=PTRNO(1)
      IF(EVENT.LE.0)T=0.0
      DO 204 I=2,NOEV
      TIMLST(I-1)=TIMLST(I)-T
      PEVLST(I-1)=PEVLST(I)
  204 PTRNO(I-1)=PTRNO(I)
      NOEV=NOEV-1
      IF(EVENT.LE.0) GO TO 11111
      TELAPS=TELAPS+T
      GO TO (10100,10200,10300,10400,10500,10600),EVENT
C
```

```
C EVENT TYPE 1..TRUCK ARRIVES AT DISPOSAL SITE
C
10100 DUMP(H)=-TELAPS
      IF(SKPDK) GO TO 10133
      DO 10101 II=1,NLD
      IF(LDBI(II).EQ.0)GO TO 10103
10101 CONTINUE
      DO 10102 IJ=1,NOEV
      IF(PEVLST(IJ).EQ.3)GO TO 10104
10102 CONTINUE
      GO TO 111
10104 CALL PUT(TIMLST(IJ),1)
      IF(UDBNT(H).EQ.0.0)UDBNT(H)=-TELAPS
      STATUS(H)=7
      GO TO 11111
10103 CONTINUE
      LDBI(II)=1
10133 CULTIM(H) = CULTIM(H) + HULTIM(H)
      CULDIS(H)=CULDIS(H)+HULDIS(H)
      IF(UDBNT(H))10105,10106,111
10105 CALL SOUP(UDBNT(H)+TELAPS,8)
      UDBNT(H)=0.0
10106 HEF = .5 * (CREW(H) * HULTIM(H)) / (TRPTON(H) * HULDIS(H))
      CALL SOUP(HEF,12)
      TRPCNT(H)=TRPCNT(H)+1.0
      CALL PUT(4.0*RANDOM(RNO)+5.0,3)
      STATUS(H)=6
      GO TO 11111
C
```

```
C EVENT TYPE 2..COLLECTION UNIT ENTERED FOR FIRST TIME
C
10200 U=NPCU(H)
      CULTIM(H)=CULTIM(H)+HULTIM(H)
      CULDIS(H)=CULDIS(H)+HULDIS(H)
      CALL CMPJOB
      CALL CAPRUL
      CALL PUT(PROP(H)*TTFPCU(H),4)
      STATUS(H)=3
      GO TO 11111
C
```

```
C EVENT TYPE 3..TRUCK LEAVES DUMP
C
10300 U=NPCU(H)
      IF(SKPDK)GO TO 10305
      DO 10301 I3=1,NLD
      IF(LDBI(I3).EQ.1)GO TO 10302
10301 CONTINUE
      GO TO 111
10302 LDBI(I3)=0
10305 X=TRPTON(H)/CAPCTY(H)
      IF(X-0.6)10307,10308,10308
10307 CALL PWFHST(X,1)
      GO TO 10309
10308 CALL PWFHST(X,2)
10309 DAYTON(H)=DAYTON(H)+TRPTON(H)
      DAYTIM(H)=DAYTIM(H)+TRPTIM(H)
      DAYDIS(H)=DAYDIS(H)+TRPDIS(H)
      CALL SOUP(TRPTON(H),2)
      CALL SOUP(TRPTIM(H),4)
      CALL SOUP(TRPDIS(H),9)
      CALL SOUP(DUMP(H)+TELAPS,7)
      IF(PROP(H).EQ.1.0)GO TO 10304
      CALL TRATIM(2)
      CALL PUT(HULTIM(H),5)
      STATUS(H)=8
      GO TO 11111
```

```
10303 CALL TRATIM(3)
      CALL PUT(HULTIM(H),6)
      STATUS(H)=9
      GO TO 11111
10304 CALL FINEXU(K,I,J)
      GO TO(10306,10303),K
10306 DSINCE(H)=J
      NPCU(H)=I
      CALL TRATIM(2)
      CALL PUT(HULTIM(H),2)
      STATUS(H)=8
      GO TO 11111
```

```
C EVENT TYPE 4.. COLLECTION UNIT IS PROP(H) FINISHED
C
10400 U=NPCU(H)
      TRPTON(H)=TRPTON(H)+PROP(H)*RFPCU(H)
      TRPTIM(H)=TRPTIM(H)+PROP(H)*TTFPCU(H)
      TRPDIS(H)=TRPDIS(H)+PROP(H)*DINPCU(H)
      IF(PROP(H).EQ.1.0)GO TO 10402
10401 CALL TRATIM(2)
      CALL PUT(HULTIM(H),1)
      STATUS(H)=5
      GO TO 11111
10402 CALL FINEXU(K,I,J)
      GO TO (10404,10401),K
10404 DSINCE(H)=J
      NPCU(H)=I
      HULTIM(H)=0.0
      STATUS(H)=4
      HULDIS(H)=0.0
      GO TO 10200
C
```

```
C EVENT TYPE 5..PARTIALLY DONE CU IS ENTERED
C
10500 U=NPCU(H)
      CULTIM(H)=CULTIM(H)+HULTIM(H)
      CULDIS(H)=CULDIS(H)+HULDIS(H)
      F=1.0-PROP(H)
      RFPCU(H)=F*RFPCU(H)
      TTFPCU(H)=F*TTFPCU(H)
      DINPCU(H)=F*DINPCU(H)
      CALL CAPRUL
      CALL PUT(PROP(H)*TTFPCU(H),4)
      STATUS(H)=3
      GO TO 11111
C
C EVENT TYPE 6..TRUCK ARRIVES AT GARAGE
C
10600 CALL SOUP(DAYTON(H),3)
      CALL SOUP(DAYTIM(H),5)
      CALL SOUP(DAYDIS(H),10)
      CALL SOUP(CULTIM(H)+HULTIM(H),6)
      CULTIM(H)=0.0
      CALL SOUP(CULDIS(H)+HULDIS(H),11)
      CULDIS(H)=0.0
      CALL SOUP(TRPCNT(H),1)
      X=TELAPS/60.0
      IF(X-11.5)10603,10604,10604
10603 CALL PWFHST(X,3)
      GO TO 10607
10604 CALL PWFHST(X,4)
10607 DO 10602 I6=1,NOEV
      IF(PTRNO(I6).EQ.H)PEVLST(I6)=-PEVLST(I6)
10602 CONTINUE
      STATUS(H)=10
      GO TO 11111
C
```

```
C ENTER HERE AT END OF MONTH
C
22001 EOM=EOM + 30
      IF(EOM.GE.122)GO TO 30000
      MOQ=MOQ + 1
      MONTH=MONTH + 1
      GO TO 22000
C
C ENTER HERE AT END OF QUARTER
C
30000 CALL REPORT
C
C RETURN TO START NEW QUARTER
C
      GO TO 300
      STOP
      END
```

```
C
      SUBROUTINE REPORT
C
C DECLARATIVES AND SPACE MANAGEMENT
C
C 1.COMMON STATEMENTS
C
      COMMON RNO,H,U,TODAY
      COMMON /TSR/NOEV,TELAPS,PEVLST(500),TIMLST(500),PTRNO(500)
      COMMON /LOCS/XCOORD,YCOORD,XCOORG,YCOORG,XCOOR(1000),YCOOR(1000)
      COMMON /CPRL/TRPTON(64),RFPCU(64),PROP(64),CAPCTY(64),TTFPCU(64)
      COMMON /CUNJB/ DIST(1000),SERV(1000),DNSTY(1000),
     A    APROD( 10),VPROD( 10),ALPHA( 10),BETA( 10),GAMMA( 10),
     B    DSINCE( 64),CREW( 64),DINPCU( 64),
     C    SEASF(12),MONTH,WESEV
      COMMON /HAUL/HULTIM(64),HULDIS(64)
      COMMON /RPRT/NUMTRK,QTR,PROBID,IF1P,IF0H
      COMMON /SCHED/DOW(1000),NFCU(64),NCUZ(64),NPCU(64),ISCHED(6,8),
     A              MSCHED(7)
      COMMON /HIST/MIST(192,18)
      COMMON /LHST/NHIST,MXAHST(15,18),LHN(3,18),C1(18),C2(18)
      COMMON /O/
     A          TCTMIN( 64),TCTMAX( 64),TCTSUM( 64),TCTSSQ( 64),
     B           TTMIN( 64), TTMAX( 64), TTSUM( 64), TTSSQ( 64),
     C           DTMIN( 64), DTMAX( 64), DTSUM( 64), DTSSQ( 64),
     F          TTIMIN( 64),TTIMAX( 64),TTISUM( 64),TTISSQ( 64),
     G          DTIMIN( 64),DTIMAX( 64),DTISUM( 64),DTISSQ( 64),
     H           CTMIN( 64), CTMAX( 64), CTSUM( 64), CTSSQ( 64),
     I          DMPMIN( 64),DMPMAX( 64),DMPSUM( 64),DMPSSQ( 64),
     J           UDMIN( 64), UDMAX( 64), UDSUM( 64), UDSSQ( 64),
     L           TDMIN( 64), TDMAX( 64), TDSUM( 64), TDSSQ( 64),
     M           DDMIN( 64), DDMAX( 64), DDSUM( 64), DDSSQ( 64),
     N           CDMIN( 64), CDMAX( 64), CDSUM( 64), CDSSQ( 64),
     O          HEFMIN( 64),HEFMAX( 64),HEFSUM( 64),HEFSSQ( 64)
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      COMMON/Z/ TRPCNT( 64),TRPTIM( 64),TRPDIS( 64),
     A          DAYTON( 64),DAYTIM( 64),DAYDIS( 64),
     B            DUMP( 64),CULTIM( 64),CULDIS( 64), UDBNT( 64)
C
C 2.TYPE DECLARATIVES
C
      INTEGER H,U,EVENT,DOW,PEVLST,PTRNO,TODAY
      INTEGER EOM,QTR,DAY
      INTEGER DIST,DSINCE,RNO
      INTEGER PROBID
      LOGICAL LOG1,LOG2,LOG3
C
C 3. ARRAYS NOT APPEARING IN COMMON STATEMENTS
C
      DIMENSION A(64,4,12)
      EQUIVALENCE (A,TCTMIN)
      DIMENSION                 MXIHST(15,18),DAYSYM(6),DAYALF(6)
      EQUIVALENCE (MXIHST,PEVLST),(DAYSYM,PEVLST(271))
      DIMENSION IFTOD(6)
      EQUIVALENCE (IFTOD,PEVLST(277))
      DIMENSION LIN(3,11),LNN(3)
C
C 4. DATA STATEMENTS
C
      DATA BLANK/2H  /
      DATA(DAYALF(I),I=1,6)/2H M,2H T,2H W,2H T,2H F,2H S/
      DATA((LIN(J,K),J=1,3),K=1, 3)/
     A54HTONS CARRIED/TRIP TONS CARRIED/DAY  PICKUP TIME /TRIP /
      DATA((LIN(J,K),J=1,3),K=4, 6)/
     B54HPICKUP TIME /DAY  HAUL   TIME /DAY  DUMP   TIME /TRIP /
      DATA((LIN(J,K),J=1,3),K=7, 9)/
     C54HWAIT   TIME /TRIP PICKUP MILES/TRIP PICKUP MILES/DAY  /
      DATA((LIN(J,K),J=1,3),K=10,11)/
     D36HHAUL   MILES/DAY  MAN*MIN/TON*MILE  /
      DATA(LNN(J),J=1,3)/18HRANGE   FREQUENCY /
```

```
C 5. FORMAT STATEMENTS
C
  601 FORMAT(46H1REFUSE COLLECTION SIMULATION REPORT FOR TRUCK,I4,
     112H, CAPACITY =,F5.2,13H TONS, CREW =,I2,9H MEN, QTRI2,
     26H, PROBI7////
     364H T H E   J O B   U N D E R T A K E N   B Y   T H I S   T R U C
     4K//
     588H COL.UNIT SVCS DENSTY XCOORD YCOORD AVPROD SDPROD  ALPHA     BE
     6TA    GAMMA   DAYS SRVCED//)
  602 FORMAT(I8,I6,5F7.2,3F9.4,6A2)
  613 FORMAT(72H . . . . . . . .D I G E S T   O F   M A I N   R E S U L T
     1 S . . . . . . . .//)
  604 FORMAT(13H DAYS WORKED I4  ,4X12HPICKUP TIME F11.3,F8.3,
     1                            2X12HAGGR COLRATE      F8.3)
  624 FORMAT(13H NUMBER TRIPSI4  ,4X12HHAUL   TIME F11.3,F8.3,
     2                            2X12HPKUP COLRATE      F8.3)
  644 FORMAT(13H MIN TRPS/DAYI4  ,4X12HDUMP   TIME F11.3,F8.3,
     3                            2X12HAVG  SPEED        F8.3)
  664 FORMAT(13H MAX TRPS/DAYI4  ,4X12HWAIT   TIME F11.3,F8.3,
     4                            2X12HEFF CAPACITY      F8.3)
  684 FORMAT(13H AVG TRPS/DAYF7.2,4X12HTOTAL  TIME F11.3,F8.3,
     5                            2X12HPCT.CAP.UTIL      F8.3)
  606 FORMAT(21X12HAVG HRS/DAY ,F11.3,9X13H AVG H-EFFCY ,F8.3)
  607 FORMAT(60H-. . .T A B L E   O F   D E T A I L E D   R E S U L T S.
     1 . .//5X7HI T E M,10X38HMINIMUM   MAXIMUM   AVERAGE   STD.DEV.//)
  608 FORMAT(1H ,3A6,4F10.3)
  609 FORMAT(6H1TRUCK,I4,89H CONT'D., FREQUENCY HISTOGRAMS.   RANGES ARE
     1 LEFT-CLOSED AND RIGHT-OPEN.   I = +INFINITY.//)
  610 FORMAT(6(1X,3A6,1X))
  611 FORMAT(6(1X,A6,I10,3X))
  612 FORMAT(1H )
  614 FORMAT(I6,3I3,4E15.8)
  615 FORMAT(I6,3I3,15I4)
  616 FORMAT(5H RNO=I12,1H$)
C
```

```
C 6. BEGIN PROGRAM
C
      LOG1=IFOH.NE.0
      LOG2=IF1P.EQ.0
      LOG3=LOG1.AND.LOG2
      DO 15 I=1,NUMTRK
      K=CREW(I)
      WRITE(6,601)I,CAPCTY(I),K,QTR,PROBID
      DO 51 II=1,6
   51 IFTOD(II)=0
      MM=NFCU(I)
      MN=NCUZ(I)+MM-1
      DO 4 U=MM,MN
      J=DIST(U)
      K=DOw(U)
      DO 3 II=1,6
      IF(ISCHED(II,K))1,2,1
    1 DAYSYM(II)=DAYALF(II)
      IFTOD(II)=1
      GO TO 3
    2 DAYSYM(II)=BLANK
    3 CONTINUE
      K=SERV(U)
    4 WRITE(6,602)U,K,DNSTY(U),XCOOR(U),YCOOR(U),APROD(J),VPROD(J),
     1 ALPHA(J),BETA(J),GAMMA(J),(DAYSYM(II),II=1,6)
      WRITE(6,613)
      DwKD=0.0
      DO 5 II=1,6
    5 IF(IFTOD(II).NE.0)DWKD=DWKD+13.0
      IDAY=DWKD
      ITRP=TCTSUM(I)
      ITMN=TCTMIN(I)
      ITMX=TCTMAX(I)
      ATPD=RND3PL(TCTSUM(I)/DWKD)
      PKPT=RND3PL(TTISUM(I))
      HAUT=RND3PL(CTSUM(I))
      DUMT=RND3PL(DMPSUM(I))
      WAIT=RND3PL(UDSUM(I))
      TOTT=PKPT + HAUT + DUMT + WAIT
      SUMT=TTISUM(I) + CTSUM(I) + DMPSUM(I) + UDSUM(I)
      AHPD=RND3PL(SUMT/(60.0*DWKD))
      PP=RND3PL(100.0*TTISUM(I)/SUMT)
      PH=RND3PL(100.0* CTSUM(I)/SUMT)
      PD=RND3PL(100.0*DMPSUM(I)/SUMT)
      PW=RND3PL(100.0* UDSUM(I)/SUMT)
      PT=PP + PH + PD + PW
      AGCOL=RND3PL(CREW(I)*SUMT/TTSUM(I))
      PUCOL=RND3PL(CREW(I)*TTISUM(I)/TTSUM(I))
      AMPH =RND3PL(60.0*CDSUM(I)/CTSUM(I))
      TSPT =TTSUM(I)/TCTSUM(I)
      EFCAP=RND3PL(TSPT)
      PCUT =RND3PL(100.0*TSPT/CAPCTY(I))
      AHEF =RND3PL(HEFSUM(I)/TCTSUM(I))
      WRITE(6,604)IDAY,PKPT,PP,AGCOL
      WRITE(6,624)ITRP,HAUT,PH,PUCOL
      WRITE(6,644)ITMN,DUMT,PD,AMPH
      WRITE(6,664)ITMX,WAIT,PW,EFCAP
      WRITE(6,684)ATPD,TOTT,PT,PCUT
      WRITE(6,606)AHPD,AHEF
      WRITE(6,607)
      DO 11 K=1,12
      GO TO(9,6,7,6,7,7,6,6,6,7,7,6),K
    7 DENOM=DWKD
      GO TO 9
    6 DENOM=TCTSUM(I)
    9 IF(.NOT.LOG2)PUNCH 614,PROBID,QTR,I,K,(A(I,J,K),J=1,4)
      IF(K.EQ.1)GO TO 11
      IF(A(I,3,K).EQ.0.0)A(I,1,K)=0.0
      A(I,3,K)=A(I,3,K)/DENOM
      PT=A(I,4,K)/DENOM-A(I,3,K)**2
      IF(PT.LT.0.0)PT=0.0
      A(I,4,K)=SQRT(PT)
      DO 10 J=1,4
   10 A(I,J,K)=RND3PL(A(I,J,K))
      WRITE(6,608)(LIN(II,K-1),II=1,3),(A(I,J,K),J=1,4)
   11 CONTINUE
      IF(LOG3)GO TO 15
      DO 13 J=1,NHIST
   13 CALL UNPWFH(I,MIST(1,J),MXIHST(1,J))
      IF(LOG1)GO TO 151
      WRITE(6,609)I
      L=-5
      MN=(NHIST+5)/6
      MM=6*MN-5
   12 L=L+6
      K=MIN0(NHIST,L+5)
      II=K-L+1
      WRITE(6,612)
      WRITE(6,610)((LHN(J,IJ),J=1,3),IJ=L,K)
      WRITE(6,610)((LNN(J),J=1,3),IJ=1,II)
      DO 14 IJ=1,15
   14 WRITE(6,611)(MXAHST(IJ,J),MXIHST(IJ,J),J=L,K)
      IF(L.NE.MM)GO TO 12
  151 IF(LOG2)GO TO 15
      DO 92 IJ=1,NHIST
   92 PUNCH 615,PROBID,QTR,I,IJ,(MXIHST(II,IJ),II=1,15)
   15 CONTINUE
      PUNCH 616,RNO

      RETURN

      END
```

```
 BLOCK DATA
 COMMON /LHST/NHIST,MXAHST(15,18),LHN(3,18),C1(18),C2(18)
 DATA NHIST/4/
 DATA (MXAHST(I,1),I=1,9)/
154H  0- 4  4- 8  8-12 12-16 16-20 20-24 24-28 28-32 32-36/
 DATA (MXAHST(I,1),I=10,15)/
136H 36-40 40-44 44-48 48-52 52-56 56-60/
 DATA (MXAHST(I,2),I=1,9)/
154H 60-64 64-68 68-72 72-76 76-80 80-84 84-88 88-92 92-96/
 DATA (MXAHST(I,2),I=10,15)/
136H 96-  100-  104-  108-  112-  116-I /
 DATA((MXAHST(I,J),I=1,15),J=3,4)/
46H 0.0- ,6H 4.5- ,6H 5.0- ,6H 5.5- ,6H 6.0- ,
56H 6.5- ,6H 7.0- ,6H 7.5- ,6H 8.0- ,6H 8.5- ,
66H 9.0- ,6H 9.5- ,6H10.0- ,6H10.5- ,6H11.0- ,
76H11.5- ,6H12.0- ,6H12.5- ,6H13.0- ,6H13.5- ,
86H14.0- ,6H14.5- ,6H15.0- ,6H15.5- ,6H16.0- ,
96H16.5- ,6H17.0- ,6H17.5- ,6H18.0- ,6H18.5-I/
 DATA((LHN(I,J),I=1,3),J=1,2)/
136HCAPACITY UTILIZED CAPACITY UTILIZED /
 DATA((LHN(I,J),I=1,3),J=3,4)/
136HLENGTH OF WORKDAY LENGTH OF WORKDAY /
 DATA (C1(I),I=1,4)/0.04,0.64,4.50,12.0/
 DATA (C2(I),I=1,4)/0.04,0.04,0.50,0.50/
  END
  SUBROUTINE CAPRUL
  COMMON RNO,H,U,TODAY
  COMMON /CPRL/TRPTON(64),RFPCU(64),PROP(64),CAPCTY(64),TTFPCU(64)
  INTEGER RNO,H
  CALL FINEXU(K,I,J)
  GO TO(1,2),K
1 PSCAP=CAPCTY(H)*RANDND(0.95,0.05,RNO)
  PROP(H)=(PSCAP-TRPTON(H))/RFPCU(H)
  IF(PROP(H).GE.1.0)PROP(H)=1.0
3 RETURN
2 IF(RFPCU(H)+TRPTON(H).GT.1.05*CAPCTY(H))GO TO 1
  PROP(H)=1.0
  GO TO 3
   END
```

```
      SUBROUTINE CMPJOB
      COMMON RNO,H,U,TODAY
      COMMON /CUNJB/ DIST(1000),SERV(1000),DNSTY(1000),
     A    APROD( 10),VPROD( 10),ALPHA( 10),BETA( 10),GAMMA( 10),
     B    DSINCE( 64),CREW( 64),DINPCU( 64),
     C    SEASF(12),MONTH,WESEV
      COMMON /CPRL/TRPTON(64),RFPCU(64),PROP(64),CAPCTY(64),TTFPCU(64)
      INTEGER H,U,DIST,DSINCE
      REAL MU
      I=DIST(U)
      D=DSINCE(H)
      MU=D*APROD(I)
      SIGMA=SQRT(D)*VPROD(I)
    1 X=RANDND(MU,SIGMA,RNO)
      IF(X.LE.D)GO TO 1
      RFPCU(H)=0.0005*(SERV(U)*SEASF(MONTH)*X)
      DINPCU(H)=SERV(U)/DNSTY(U)
      TTFPCU(H)=
     1 WESEV*(ALPHA(I)*SERV(U) + (GAMMA(I)*DINPCU(H))/BETA(I))
      RETURN
       END
      SUBROUTINE FINEXU(IR,IU,ID)
      COMMON RNO,H,U,TODAY
      COMMON /SCHED/DOW(1000),NFCU(64),NCUZ(64),NPCU(64),ISCHED(6,8),
     A               MSCHED(7)
      INTEGER H,DOW
      INTEGER TODAY
      IR=2
      IF=NPCU(H)+1
      MM=NFCU(H)+NCUZ(H)
      IF(IF.EQ.MM)GO TO 2
      MM=MM-1
      DO 1 I=IF,MM
      IG=DOW(I)
      J=ISCHED(TODAY,IG)
      IF(J.NE.0)GO TO 3
    1 CONTINUE
    2 RETURN
    3 IR=1
      IU=I
      ID=J
      GO TO 2
       END
      SUBROUTINE HISTOG(A,N,K)
      DIMENSION A(N)
      COMMON RNO,H,U,TODAY
      INTEGER H,U,TODAY
      X=RANDOM(RNO)
      Y=0.0
      DO 1 I=1,N
      Y=Y+A(I)
      IF(Y.GE.X)GO TO 2
    1 CONTINUE
      K=N
      GO TO 3
    2 K=I
    3 RETURN
       END
      SUBROUTINE PUT(T,I)
      COMMON RNO,H,U,TODAY
      COMMON /TSR/NOEV,TELAPS,PEVLST(500),TIMLST(500),PTRNO(500)
      INTEGER PEVLST,PTRNO
      INTEGER H,U,TODAY
      K=1
      IF(NOEV.EQ.0)GO TO 2
      DO 1 J=1,NOEV
      IF(T-TIMLST(J))3,1,1
    1 CONTINUE
      K=NOEV+1
    2 TIMLST(K)=T
      PEVLST(K)=I
      PTRNO(K)=H
      NOEV=NOEV+1
      RETURN
    3 K=J
      DO 4 M=J,NOEV
      L=NOEV+J-M
      TIMLST(L+1)=TIMLST(L)
      PEVLST(L+1)=PEVLST(L)
    4 PTRNO(L+1)=PTRNO(L)
      GO TO 2
       END
      SUBROUTINE SOUP(X,K)
      INTEGER H
      COMMON RNO,H,U,TODAY
      COMMON /O/
     A          TCTMIN( 64),TCTMAX( 64),TCTSUM( 64),TCTSSQ( 64),
     B           TTMIN( 64), TTMAX( 64), TTSUM( 64), TTSSQ( 64),
     C           DTMIN( 64), DTMAX( 64), DTSUM( 64), DTSSQ( 64),
     F          TTIMIN( 64),TTIMAX( 64),TTISUM( 64),TTISSQ( 64),
     G          DTIMIN( 64),DTIMAX( 64),DTISUM( 64),DTISSQ( 64),
     H           CTMIN( 64), CTMAX( 64), CTSUM( 64), CTSSQ( 64),
     I          DMPMIN( 64),DMPMAX( 64),DMPSUM( 64),DMPSSQ( 64),
     J           UDMIN( 64), UDMAX( 64), UDSUM( 64), UDSSQ( 64),
     L           TDMIN( 64), TDMAX( 64), TDSUM( 64), TDSSQ( 64),
     M           DDMIN( 64), DDMAX( 64), DDSUM( 64), DDSSQ( 64),
     N           CDMIN( 64), CDMAX( 64), CDSUM( 64), CDSSQ( 64),
     O          HEFMIN( 64),HEFMAX( 64),HEFSUM( 64),HEFSSQ( 64)
      DIMENSION A(64,4,12)
      EQUIVALENCE (A,TCTMIN)
      A(H,3,K)=A(H,3,K)+X
      A(H,4,K)=A(H,4,K)+X**2
      IF(X.LT.A(H,1,K))A(H,1,K)=X
      IF(X.GT.A(H,2,K))A(H,2,K)=X
      X=0.0
      RETURN
       END
      SUBROUTINE TRATIM(J)
      COMMON RNO,H,U,TODAY
      COMMON /LOCS/XCOORD,YCOORD,XCOORG,YCOORG,XCOOR(1000),YCOOR(1000)
      COMMON /HAUL/HULTIM(64),HULDIS(64)
      INTEGER H,U,TODAY
      GO TO(1,2,8),J
    1 ABSIS=XCOORG
      ORDIN=YCOORG
      GO TO 3
    8 Y=ABS(XCOORD-XCOORG)+ABS(YCOORD-YCOORG)
      GO TO 31
    2 ABSIS=XCOORD
      ORDIN=YCOORD
    3 Y=ABS(ABSIS-XCOOR(U))+ABS(ORDIN-YCOOR(U))
   31 IF(Y.EQ.0.0)GO TO 5
      D=2.0*Y-5.5
      IF(D)4,4,6
    4 SS = 6.0
      GO TO 7
    6 SS = 6. - D / (.4448 + .2646 * D)
    7 S = SS * RANDND(1.,.1,RNO)
      IF(S)7,5,5
    5 HULDIS(H)=Y
      HULTIM(H)=Y*S
      RETURN
       END
```

```
SUBROUTINE PwFHST(X,I)
INTEGER H
COMMON RNO,H,U,TODAY
COMMON /HIST/ MIST(3,64,18)
COMMON /LHST/ JAZZ(325),C1(18),C2(18)
DIMENSION INCR(5)
DATA INCR /268435456,2097152,16384,128,1/
II= I
NHF = (X - C1(II)) / C2(II)
NHF = MAX0(0,MIN0(14,NHF + 1))
Nw= NHF/5+1
NF= NHF-5*NW+6
MIST(NW,H,II)= MIST(NW,H,II)+INCR(NF)
RETURN
 END
```

```
  FUNCTION RND3PL(X)
  I= 1000.*(X+.0005)
  Y= I
  RND3PL= Y/1000.
  RETURN
   END

  SUBROUTINE UNPWFH(H,NAMHST,IV)
  INTEGER H
  DIMENSION NAMHST(3,64),IV(5,3)
  DO 1 I= 1,3
  J= NAMHST(I,H)
  DO 1 L= 1,5
  LB= 6-L
  K= J
  J= K/128
1 IV(LB,I) = K - 128 * J
  RETURN
   END
```

```
      FUNCTION RANDND(XMU,SIGMA,IR)
      LOGICAL SWITCH
      DATA SWITCH /.TRUE./
      EQUIVALENCE (F,Y),(X2,U2)
      SWITCH= .NOT.SWITCH
      IF(SWITCH) GO TO 2
      F= SQRT(-2.*ALOG(RANDOM(IR)))
      U2= 6.2831853*RANDOM(IR)
      X1= F*SIN(U2)
      X2= F*COS(U2)
      Y= X1
    1 RANDND = SIGMA * Y + XMU
      RETURN
    2 Y = X2
      GO TO 1
       END

      FUNCTION RANDOM(SEED)
C
C NOTE .. THE VALIDITY OF THIS ROUTINE IS DEPENDENT ON THE 35-BIT WORD
C         LENGTH AND THE EXCESS-128 EXPONENT LOGIC OF THE 700 SERIES
C         COMPUTERS.  DON'T SAY YOU WEREN'T WARNED.
C
      INTEGER ALPHA,SEED,EXPON
      DATA ALPHA,EXPON/30517578125,0200000000000/
      EQUIVALENCE (X,I),(EXPON,XPON)
      SEED = SEED * ALPHA
      I = SEED / 256 + EXPON
      RANDOM = X + XPON
      RETURN
      END
```

GLOSSARY OF NAMES AND TERMS FOR APPENDIX 2-A

A. Variables

APROD - mean of refuse production in lbs/serv/day.

CREW - number of men on truck.

CULDIS - cumulative haul distance.

CULTIM - cumulative haul time.

DAYDIS - total distance during day.

DAYTIM - total time during day.

DAYTON - total tons hauled during day.

DINPCU - distance inside present CU.

DIST - number of district to which CU belongs.

DNSTY - service density in services/mile.

DSINCE - days since last collection of CU NPCU(I).

DUMP - temporary counter to count time (min.) spent by truck at disposal site.

HULDIS - haul distance.

HULTIM - haul time.

LDBI - tells whether unloading berth is being used.

NOEV - number of events.

NPCU - number of collection units presently being serviced.

PEVLST - list of pending events.

PROP - proportion of refuse present in this CU that can be loaded.

PSCAP - pseudo-capacity of truck.

PTRNO - number of truck involved in pending event.

RFPCU - refuse in present CU.

SEASF - seasonal factor (month).

SERV - number of services.

SKPDK - logical variable (skip dock) see page 2-190.

TELAPS - time elapsed (master daily time counter).

TRPCNT - trip count.

TTFPCU - time to finish present CU.

UDBNT - unloading dock bottleneck time.

VPROD - standard dev. of refuse production in lbs/serv/day

WESEV - weather severity.

XCOORN, YCOORN - physical coordinates of

N = blank: CU's

N = D : Disposal site

N = G : Garage

B. Status Code

Status Code	
1	Truck not working today.
2	Truck enroute from garage to first CU of day.
3	Truck collecting refuse (from NPCU(I)).
4	Truck finished with collection unit and is proceeding to next.
5	Truck has finished a load and is traveling to disposal.
6	Truck is at disposal site and is dumping.
7	Truck is at disposal site and is waiting for vacant unloading dock.
8	Truck has dumped load and is traveling to next-load area.
9	Truck has dumped last load of the day and is enroute to garage.
10	Truck at garage and through for day.

REFERENCES FOR PREFACE

[Pr 1] *An Analysis of Refuse Collection and Sanitary Landfill Disposal*, Sanitary Engineering Research Project, Technical Bulletin No. 8, University of California, Berkeley, December 1952.

[Pr 2] Deininger, R. A., *Water Quality Management: The Planning of Economically Optimal Pollution Control Systems*, Ph.D. Dissertation, Northwestern University, Evanston, Illinois, June 1965.

[Pr 3] Galler, W. S., *Optimization Analysis for Biological Filters*, Ph.D. Dissertation, Northwestern University, Evanston, Illinois, June 1965.

[Pr 4] _______________, and Gotaas, H. B., "Optimization Analysis for Bilogical Filter Design," *J. Sanitary Engr. Div. A.S.C.E*, V. 92, No. SA 1, pp. 163-182, February 1966.

[Pr 5] Hufschmidt, M. M. and Fiering, M. B., *Simulation Techniques for the Design of Water Resources Systems*, Harvard University Press, Cambridge, 1966.

[Pr 6] Loucks, D. P., *A Probablistic Analysis of Waste Water Treatment Systems*, Ph.D. Dissertation, Cornell University, Ithaca, N. Y., September 1965.

[PR 7] _______________, ReVelle, C. S., and Lynn, W. R., "Linear Programming Models for Water Pollution Control," *Management Science*, V. 14, No. 4, pp. B166-B181, December 1967.

[Pr 8] Lynn, W. R., Logan, J. A., and Charnes, A., "Systems Analysis for Planning Wastewater Treatment Plants," *J. Water Pollution Control Federation*, V. 34, pp. 565-581, June 1962.

[Pr 9] ______________, and Charnes, A., "The Dyadic Computation of a Generalized Network Model of a Sewage Treatment System," ONR Research Memorandum 19, Northwestern University, Technological Inst., October, 1959.

[Pr 10] ______________, Ph.D. Dissertation, *Process Design and Financing of Sewage Treatment Works*, Northwestern University, Evanston, Illinois, June 1963.

[Pr 11] ______________, Appendix D of *Solid Wastes Research Needs*, APWA Research Foundation, Project No. 113, Chicago, Illinois, May 1962.

[Pr 12] Maas, A. *et al.*, *Design of Water Resources Systems*, Harvard University Press, Cambridge, Massachussetts, 1962.

[Pr 13] Sheaffer, J. R. *et al.*, *Refuse Disposal Needs and Practices in Northeastern Illinois*, Technical Report No. 3, Northeastern Illinois Metropolitan Area Planning Commission, Chicago, Illinois, June 1963.

[Pr 14] Smallwood, C., Jr., Galler, W. S., and Anderson, C. A., "Evaluation of Alternatives in Refuse Disposal-Progress Report, First Year (January 1 - December 31, 1967)," USPHS Demonstration Grant 1-D01-U1-0050-01, North Carolina State University at Raleigh, Raleigh, January 1968.

[Pr 15] de Tocqueville, Alexis, *La Démocratie en Amérique*, 1835. This book is a journal and series of essays reporting on the travels in America of this distinguished French statesman and man of letters. In it he comments on the social and political institutions of the young republic and describes in great detail the unspoiled wonders of the American wilderness.

REFERENCES FOR PART I

A. Mathematics, Economics, and Location Theory

[A1] Armour, G. C., and Buffa, E.S., "A Heuristic Algorithm and Simulation Approach to Relative Location of Facilities," Man. Sci., V. 9, pp. 294-309, (1963).

[A2] Bellman, R., "An Application of Dynamic Programming to Location-Allocation Problems," SIAM Review, V. 7, pp. 126-128, (1965).

[A3] Baumol, W. J., and Wolfe, P., "A Warehouse Location Problem," Op. Res., V. 6, pp. 252-263, (1958).

[A4] Charnes, A., Quon, J. E., and Wersan, S. J., "Location-Allocation Problems in the ℓ_1-Metric," Joint CORS-ORSA 25th Nat'l Meet., Montreal, Que., May 1964.

[A5] Cooper, L., "Location-Allocation Problems," Op. Res., V. 11, pp. 331-343, (1963).

[A6] _______________, "Heuristic Methods for Location-Allocation Problems," SIAM Review, V. 6, pp. 37-53, (1964).

[A7] Drysdale, J. K., and Sandiford, P. J., "A Method for Locating Warehouses," 30th Nat'l Meet. ORSA, Durham, N. C., October 1966.

[A8] Efroymson, M. A., and Ray, T. L., "A Branch-Bound Algorithm for Plant Location," Op. Res., V. 14, pp. 361-368, (1966).

[A9] Eilon, S., and Deziel, D. P., "Siting a Distribution Centre, an Analogue Computer Application," Man. Sci., V. 12, pp. B245-B254, (1966).

[A10] Eisemann, K., "The Optimum Location of a Center," Problem No. 62-11 in "Problems and Solutions," M. Klamkin, Ed., *SIAM Review*, V. 4., pp. 394-395, (1962).

[A11] Francis, R. L., "On the Location of Multiple New Facilities with Respect to Existing Facilities," *J. Ind. Eng.*, V. 15, pp. 106-107,

[A12] Frank, H., "Optimum Locations on a Graph with Probabilistic Demands," *Op. Res.*, V. 14, pp. 409-421, (1966).

[A13] Friedrich, C. J., *Alfred Weber's Theory of the Location of Industries*, U. Chicago Press, Chicago, Ill., 1929.

[A14] Garett, J. W., and Plyter, N. V., "The Optimal Assignment of Facilities to Locations by Branch and Bound," *Op. Res.*, V. 14, pp. 210-252, (1966).

[A15] Hakimi, S. L., "Optimum Locations of Switching Centers and the Absolute Centers and Medians of a Graph," *Op. Res.*, V. 12, pp. 450-459, (1964).

[A16] Heller, I., "On the Problem of Shortest Path Between Points," I & II (Abstract), *Bull. A. Math. Soc.*, 59, 6, (November, 1953).

[A17] Hillier, F. S., "Quantitative Tools for Plant Layout Analysis," *J. Ind. Eng.*, V. 14, pp. 33-40, (1963).

[A18] Isard, W., *Location and Space Economy*, Technology Press, M.I.T., Cambridge, Mass., 1956.

[A19] Kantorovitch, L., "On the Translocation of Masses," *Man. Sci.*, V. 5, pp. 1-4, (1958).

[A20] Koopmans, T. C. and Beckmann, M., "Assignment Problems and the Location of Economic Activity," *Econometrica*, V. 25, pp. 53-76, (1957).

[A21] Kuehn, A. A., and Hamburger, M. J., "A Heuristic Program for Locating Warehouses," *Man. Sci.*, V. 9, pp. 643-666, (1963).

[A22] Kuhn, H. W., and Kuenne, R. E., "An Efficient Algorithm for the Numerical Solution of the Generalized Weber Problem in Spatial Economics," *J. Reg. Sci.*, V. 4, pp. 21-33, (1962).

[A23] McIlroy, M. D., "Transportation Problems with Distributed Loads," abstract in *Recent Advances in Mathematical Programming*, R. L. Graves and P. Wolfe, Eds., McGraw-Hill, New York, (1963).

[A24] Maranzana, F. E., "On the Location of Supply Points to Minimize Transportation Costs," *IBM Systems J.*, pp. 129-135, June 1963.

[A25] Marschak, T., "Centralization and Decentralization in Economic Organizations," *Econometrica*, V. 27, pp. 399-429, (1959).

[A26] Noble, B., "Optimum Location Problems," ch. 2 of *Applications of Undergraduate Mathematics in Engineering*, pp. 13-33, Macmillan Co., New York, (1967).

[A27] Palermo, F. P., "A Network Minimization Problem," *IBM J.*, V. 5, pp. 335-337, (1961). See [A30].

[A28] Prager, W., "A Generalization of Hitchcock's Tran Transportation Problem," *J. Math. Physics*, V. 36, pp. 99-106, (1957-1958).

[A29] ________________, "Numerical Solution of the Generalized Transportation Problem," *Naval Res. Log. Q.*, V. 4, pp. 253-261, (1957).

[A30] Tideman, M., "Comment on 'A Network Minimization Problem'," *IBM J.*, V. 6, pp. 259, (1962).

[A31] Tutte, W. T., "On Hamiltonian Circuits," *London Math. Soc. J.*, XXI, Part 2, 82, pp. 98-101, (1946).

[A32] Verblinsky, S., "On the Shortest Path Through a Number of Points," *Proc. Am. Math. Soc.*, II, 6 (Dec. 1951).

[A33] Vergin, R. C. and Rogers, J. D., "An Algorithm and Computational Procedure for Locating Economic Facilities," *Man. Sci.*, V. 13, pp. B240-B251, (1967).

[A34] Wersan, S. J., Quon, J. E., and Charnes, A., "Systems Analysis of Refuse Collection and Disposal Practices," *1962 Yearbook A.P.W.A.*, Chicago, Illinois, (1962).

B. *Computer Technology: Backplane Wiring*

[B1] Glaser, R. H., "A Quasi-Simplex Method for Designing Suboptimum Packages of Electronic Building Blocks," *1959 Computer Applications Symposium*, Armour Research Foundation, Illinois Institute of Technology, Chicago, Illinois.

[B2] Kurtzberg, J. M., "Algorithms for Backplane Formulation," *Proc. O.N.R. Symposium on Microelectronics and Large Systems*, Washington, D. C., 18 November 1964.

[B3] ______________, "Backboard Wiring Algorithms for the Placement and Connection Order Problems," *Burroughs Corp. Tech. Rep. TR60-40* (Company Confidential), 28 June 1960.

[B4] ______________, "Computer Mechanization of Design Procedures," *Proc. 6th Annual Conf. Am. Inst. Ind. Eng.*, Detroit, Michigan, 15 October 1964.

[B5] ______________, and Seward, J., "Program for Star Cluster Wiring of Backplanes," *Burroughs Corp. Internal Report*, January 1964.

[B6] Loberman, M., and Weinberger, A., "Formal Procedures for Connecting Terminals with a Minimal Total Wire Length," *J.A.C.M.* V. 4, pp. 428-427, (1957).

[B7] Rutman, R. A., "An Algorithm for Placement of Interconnected Elements Based on Minimum Wire Length," *AFIPS Proc*., 1964 Spring Joint Computer Conf., April 1964.

[B8] Steinberg, L., "The Backboard Wiring Problem: A Placement Algorithm," *SIAM Review*, V. 3, pp. 37-42, (1961).

C. *Integer Programming*

[C1] Balas, E., "An Additive Algorithm for Solving Linear Programs with Zero-one Variables," *Op. Res*., V. 13, pp. 517-546, (1965).

[C2] Balinski, M. L., "Integer Programming: Methods, Uses, Computation," *Man. Sci*., V. 12, pp. 253-313, (1965).

[C3] Beale, E. M. L., "A Method of Solving Linear Programming Problems When Some but Not all of the Variables Must Take Integral Values," *Statistical Techniques Research Group*, *Technical Report No. 19*, Princeton University, (1958).

[C4] Ben Israel, A., and Charnes, A., "On Some Problems of Diophantine Programming," *Cahiers du Centre l'Etudes de Récherce Opérationelle*, V. 4, pp. 215-280, Brussels, 1962.

[C5] Benders, J. F., Catchpole, A. R., and Kuiken, L. C., "Discrete Variable Optimization Problems," Paper presented to the *Rand Symposium on Mathematical Programming*, Santa Monica, 1959.

[C6] Charnes, A., and Cooper, W. W., *Management Models and Industrial Applications of Linear Programming*, John Wiley, New York, 1961.

[C7] Dantzig, G. B., "Discrete Variable Extremum Problems," Op. Res., V. 5, pp. 266-277, (1957).

[C8] Elmaghraby, S. E., "An Algorithm for the Solution of the Zero-One Problem of Integer Linear Programming," Doctoral Dissertation, Department of Industrial Administration, Yale University, May, 1963.

[C9] Fleishmann, B., "Computational Experience with the Algorithm of Balas," Op. Res., V. 15, pp. 153-155, (1967).

[C10] Gilmore, P. C., and Gomery, R. E., "A Linear Programming Approach to the Cutting-Stock Problem," Op. Res., V. 9, pp. 849-859, (1961).

[C11] _______________, and _______________, " A Linear Programming Approach to the Cutting-Stock Problem, Part II," Op. Res., V. 11, pp. 863-888, (1963).

[C12] Glover, F., "A Bound Escalation Method for the Solution of Integer Programs," Cahiers (See [C4].), V. 6, (1964).

[C13] _______________, "A Multiphase Dual Algorithm for the Zero-One Integer Programming Problem," Op. Res., V. 13, pp. 879-919, (1965).

[C14] _______________, and Zionts, S., "A Note on the Additive Algorithm," Op. Res., V. 13, pp. 546-549, (1965).

[C15] Gomery, R. E., "Outline of an Algorithm for Integer Solutions to Linear Programs," Bull. A.M.S., 64, 3(1958).

[C16] _______________, "Essentials of an Algorithm for Integer Solutions to Linear Programs," in Recent Advances in Mathematical Programming, R. L. Graves and P. Wolfe, Eds., McGraw-Hill, New York, 1963.

[C17] _______________, "An All-Integer Programming Algorithm," Ch. 13 of *Industrial Scheduling*, J. R. Muth and G. L. Thompson, Eds., Prentice-Hall, Englewood Cliffs, 1963.

[C18] _______________, and Hoffman, A. J., "On the Convergence of an Integer Programming Process," *Naval Research Logistics Quarterly*, V. 10, pp. 121-124, (1963).

[C19] Haldi, J., and Isaacson, L. M., "A Computer Code for Integer Solutions to Linear Programs," "*Op. Res.*, V. 13, pp. 946-959, (1965).

[C20] Land, A. H., and Doig, A. G., "An Automatic Method of Solving Discrete Programming Problems," *Econometrica*, V. 28, pp. 497-520, (1960).

[C21] Lemke, C. E., and Spielberg, K., "Direct Search Zero-One Integer and Mixed Integer Programming," *IBM 39.008*, *Report No. 3*, June 1966.

[C22] Markowitz, H. M., and Manne, A. S., "On the Solution of Discrete Programming Problems," *Econometrica*, V. 25, pp. 84-110, (1957).

[C23] Martin, G. T., "An Accelerated Euclidean Algorithm for Integer Linear Programming," in *Recent Advances* (See [C15].).

[C24] Petersen, C.C., "Computational Experience with Variants of the Balas Algorithm Applied to the Selection of R & D Projects," *Man. Sci.*, V. 13, pp. 736-745, (1967).

[C25] Thompson, G. L., "The Stopped Simplex Method, Part I," *Revue Francaise de Recherche Opérationelle*, V. 8, pp. 159-182, (196).

[C26] _______________, "The Stopped Simplex Method, Part II," *Revue Francaise de Recherche Opérationelle*, V. 9, pp. _______, (1965).

[C27] Tucker, A. W., "On Directed Graphs and Integer Programs," *Princeton-IBM Mathematical Research Projects, Technical Report, 1960*.

[C28] Walker, R. J., "An Enumerative Technique for a Class of Combinatorial Problems, *Combinatorial Analysis*, *Proc. of Symposia in Appl. Math.*, R. Bellman and M. Hall, Jr., Eds., V. 10, pp. 91-94, A.M.A., Providence, R.I., 1960.

[C29] Wersan, S. J., "Computer Implementation of Balas' Algorithm for Zero-One Linear Programs," presented at ACM-SIGMAP *Colloquium on Branch and Bound Methods*, Princeton, N. J., January 29-31, 1967.

[C30] _______________, "BALGOR: A Program for Solving Linear Programs in Zero-One Variables," *Aerospace Corporation Report No. TR-0158 (S3816-10)-1*, San Bernardino, California, September 1967.

[C31] Young, R. D., "A Primal (All-Integer) Integer Programming Algorithm," *J. of Res. of the N.B.S. - B. Mathematics and Mathematical Physics*, V. 69B, No. 3, pp. 213-250, (July-September 1965).

[C32] Zionts, S., Thompson, G. L., and Tonge, F. M., "Techniques for Removing Nonbinding Constraints and Extraneous Variables from Linear Programming Problems," Carnegie Inst. of Technology, Graduate School of Ind. Admin., November, 1964.

D. *Bibliographies and Encyclopedias*

[D1] *Encyklopadie der Mathematischen Wissenschaffen*, V. 3, Part 1, B. G. Teuber, Leipzig, 1907.

[D2] *Selected Bibliography on Location Theory*, (mimeographed), Northwestern University, Transportation Center Library, Evanston, Illinois, 10 December 1963.

[D3] *Industrial Location Bibliography*, Real Estate Research Program, Division of Research, Graduate School of Business Administration, University of California, Los Angeles, July 1959.

REFERENCES FOR PART II

[1] An Analysis of Refuse Collection and Sanitary Landfill, Technical Bulletin No. 8, Series 37, Sanitary Engineering Research Project, University of California, Berkeley, December, 1952.

[2] "Fresno Region Solid Waste Management Study," Vols. I, II, and III, a report to California Department of Public Health, Report No. 3413, by Aerojet General Corporation and Engineering Science, Inc., Azusa, California, June, 1967.

[3] Modeling and Simulation in Operations Research, compendium of lecture notes for two week course of same name, Engineering Summer Conferences, University of Michigan, Ann Arbor, Michigan, 1960.

[4] Municipal Refuse Disposal, prepared by the Committee on Refuse Disposal, American Public Works Association, A.P.W.A. Research Foundation, 1961.

[5] Refuse Collection Practice, second edition, prepared by the Committee on Refuse Collection, American Public Works,Association, A.P.W.A., Research Foundation, 1958.

[6] Solid Waste Research and Development, proceedings of the Engineering Foundation Research Conference, University School, Milwaukee, Wisconsin, July 24-28, 1967. The individual papers referred to in this dissertation are:

McGaubey, P. H., "Goals of the Conference," Keynote speech. Contrasts research methodologies.

Weaver, L., "Solid Wastes: A New Test for Research Engineering," Keynote speech. Statement of crisis; significance of Solid Wastes Disposal Act of 1965.

A1 Golueke, C. G., "Comprehensive Studies of Solid Waste Management."

A2 Spradlin, B. C., "The System of Dynamics of Solid Waste Management and Control."

A3 Mitchell, R. E., Bowerman, F. R., and Walsh, T. E., "A Systemized Approach to Urban-Rural Solid Waste Management."

A7 Galler, W. S., "Study and Investigation of Solid Waste Disposal, City of Raleigh, North Carolina."

A11 Dudley, R. H., and Hekemian, K. K., A Systems Approach to Solid Waste Management."

D1 Betz, J. M., "Application of EDP and OR Techniques."

D2 Kruse, C. W., Liebman, J. C., and Truitt, M. M., "Optimal Policies for Solid Waste Collection."

D4 Zandi, I., "Solid Waste Pipeline."

D5 First, M. W., et. al., "Shipborne Municipal Incineration."

F3 Hickman, H. L., Jr., and Kennedy, J. C., "Studies of Characteristics of Municipal and Residential Solid Waste."

F4 ________________, and ________________, "Seasonal Variation in Municipal Solid Waste Output."

F5 Summary of [14].

F6 Ecke, D. H., Linsdale, D. W., Rogers, P. A., and Bellenger, G., "Fly and Economic Evaluation of Urban Garbage Systems."

[7] Bowerman, F. R., "Municipal Refuse Transfer Stations," *1962 Yearbook of the American Public Works Association*, Chicago, Illinois, December 1962, pp. 191-194. Presented at A.P.W.A. Congress, New Orleans, Louisana, October, 1962.

[8] Curtis, W. H., and Martin, R. M., "An Information Retrieval System for Urban Areas, (Refuse Scheduling Analysis)," Technical Report, City of Vancouver, B. C., February, 1965.

[9] Golueke, C. G., and McGauhey, P. H., "Comprehensive Studies of Solid Wastes Management," Sanitary Engineering Research Laboratory, Reports No. 17-36, University of California, Berkeley, California.

[10] Guetzkow, H., Ed., *simulation in social sciences: readings*, Prentice-Hall, Englewood Cliffs, N. J., 1963.

[11] Orcutt, G., *et. al.*, *Microanalysis of Socionomic Systems: A Simulation Study*, Harper and Row, New York, 1966.

[12] Quon, J. E., Charnes, A., and Wersan, S. J., "Location of Disposal Sites," First Progress Report under Research Grant EF 00355-01 (Division of Environmental Engineering and Food Protection Bureau of State Services U.S.P.H.S.,) Northwestern University Technological Institute, May 20, 1963.

[13] ______________, ______________, ______________ "Simulation and Analyses of A Refuse Collection System," *Journal of the Sanitary Engineering Division, ASCE*, Vol. 91, No. SA5, Proc. Paper 4491, October, 1965, pp. 17-36. Presented at ORSA Symposium on Operations Research and the Quality of the Environment, New York, N. Y., May 30, 1967.

[14] ______________, Tanaka, M., and Charnes, A., "Refuse Quantities and the Frequency of Service," *Journal of the Sanitary Engineering Division, ASCE*, Vol. 94, No. SA2, Proc. Paper 5917, April, 1968, pp. 403-420.

[15] ________, ________, and Wersan, S. J., "Simulation Model of Refuse Collection Policies," *Journal of the Sanitary Engineering Division, ASCE*, Vol. 95, SA3, pp. 575-592, June 1969.

[16] Wersan, S. J., Eisen, R., and Charnes, A., "Computer Simulation of a Refuse Collection System," *Systems Research Memorandum No. 101*, Northwestern University, April, 1964.

[17] ________, Quon, J. E., and Charnes, A., "Location-Allocation Problems in the ℓ_1-Metric," presented at the CORS/ORSA Meeting, Montreal, Quebec, May 28, 1964.

[18] ________, ________, and ________ "Systems Analysis of Refuse Collection and Disposal Practices," *1962 Yearbook of the American Public Works Association*, Chicago, Illinois, December 1962, pp. 195-211. Presented at A.P.W.A. Congress, New Orleans, Louisana, October 3, 1962.